"Formulações estendidas para o problema do caminho mais curto com restrições de salto"

Luis Gouveia, FC/UL-CIO

Ciência 2010

Hard to find if Problem is NP-Hard

$$\begin{aligned} \min\ & cx \\ s.a\ & Ax \geq b \\ & x \geq 0 \ e \ \text{inteiro} \end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned} \min\ & cx \\ s.a\ & Conv\{x : Ax \geq b \\ & \qquad x \geq 0 \ e \ \text{inteiro}\} \end{aligned}$$

Hard to find if Problem is NP-Hard

$$
\begin{aligned}
&\min \ cx \\
&s.a \quad Ax \geq b \\
&\qquad\quad x \geq 0 \ e \ \text{inteiro}
\end{aligned}
$$

$$
\begin{aligned}
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&s.a \quad Conv\{x : \ Ax \geq b \\
&\qquad\qquad\qquad\qquad x \geq 0 \ e \ \text{inteiro}\}
\end{aligned}
$$

$$
\begin{aligned}
&\min \ cx \\
&s.a \quad A1x \geq b1 \\
&\qquad\quad A2x \geq b2 \\
&\qquad\quad x \geq 0 \ e \ \text{inteiro}
\end{aligned}
$$

Hard to find if Problem is NP-Hard

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$$\begin{aligned} &\min \ cx \\ &s.a \ Conv\{x : \ A1x \geq b \\ &\qquad\qquad x \geq 0 \ e \ \text{inteiro}\} \\ &\qquad Conv\{x : \ A2x \geq b \\ &\qquad\qquad x \geq 0 \ e \ \text{inteiro}\} \end{aligned}$$

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$$\min \ cx$$
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Projection

$$Bx + Cw \geq d$$
$$x, w \geq 0$$

Path Reformulations

$$
\begin{aligned}
&\min \ cx \\
&s.a \quad A1x \geq b1 \\
&\qquad A2x \geq b2 \\
&\qquad x \geq 0 \ e \ \text{int}\,eiro
\end{aligned}
$$

$$
\begin{aligned}
&A2x \geq b2 \\
&x \geq 0 \ e \ \text{int}\,eiro
\end{aligned}
$$

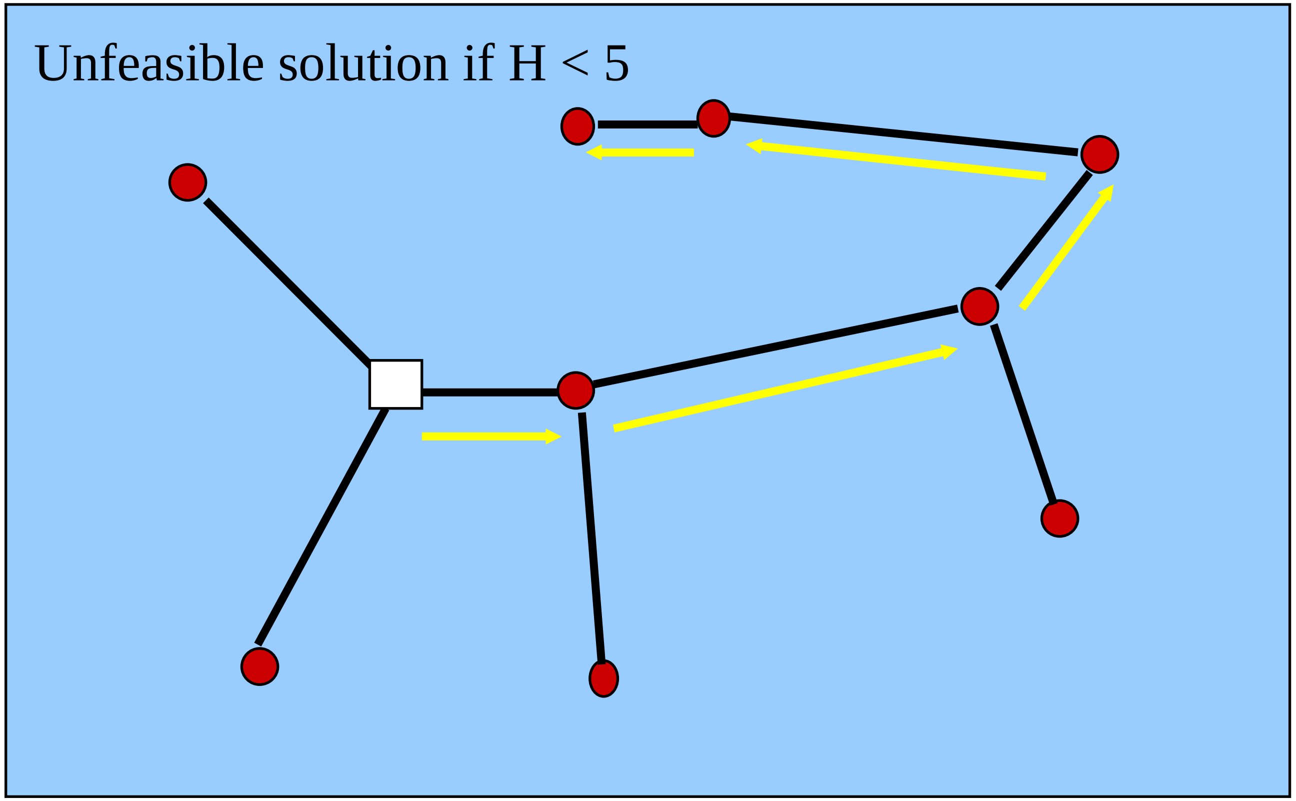
Path in an Expanded Graph
Usual Exact Formulation in Expanded Graph
Expanded Graph of Polynomial Size

Extended Exact Formulation in Original Graph
Compact Formulation

$$
\begin{aligned}
&Bx + Cw \geq d \\
&x, w \geq 0
\end{aligned}
$$

Hop-Constrained Spanning Tree =

Spanning Tree +
for each j ≠ 0, nº of arcs in the path from node 0 to node j is ≤ H



NP-Hard

Special case of Location problem (H=2)

Hop Constrained Spanning Trees

min $$\sum_{i=0}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$$

s. a. $$\sum_{i=0}^{n} x_{ij} = 1 \qquad j = 1,\ldots,n$$

$\{(i,j): i=0,\ldots,n; j = 1,\ldots,n;\ y^k_{ij} = 1\}$ contains a path from 0 to k with at most H hops $\forall$k

"contains" ?!

$$y_{ij}^{k} \leq x_{ij} \qquad i = 0,\ldots,n;\ j = 1,\ldots,n$$

$$x_{ij} \in \{0,1\} \qquad i = 0,\ldots,n;\ j = 1,\ldots,n$$

HC-Path(k)

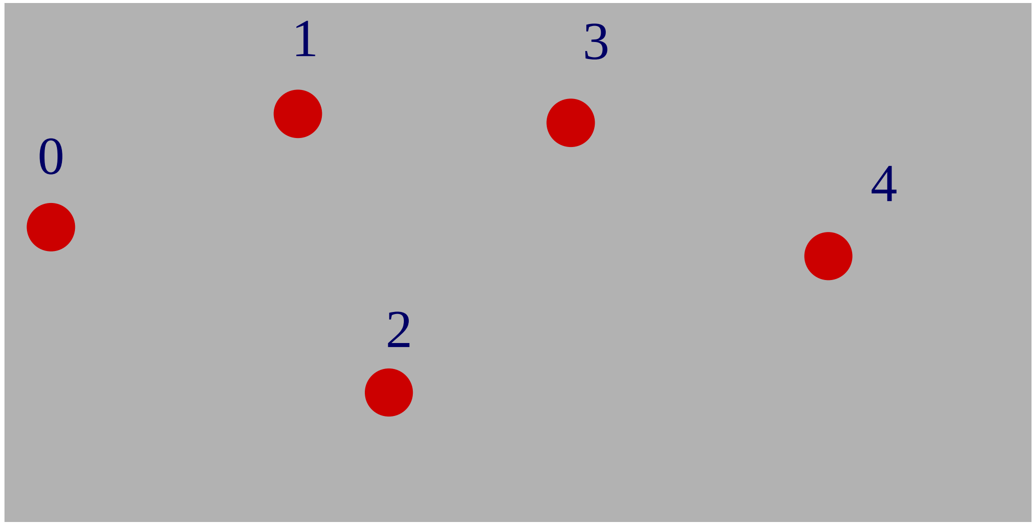
$$\sum_{i=1}^{n} f_{ij}^{k} - \sum_{i=1}^{n} f_{ji}^{k} = \begin{cases} -1 & se \quad j=1 \\ 0 & se \quad j=2,\dots,n;\ j\neq k \\ 1 & se \quad j=k \end{cases}$$

$$\sum_{i=0}^{n}\sum_{j=1}^{n} f_{ij}^{k} \leq H \qquad \text{!?}$$

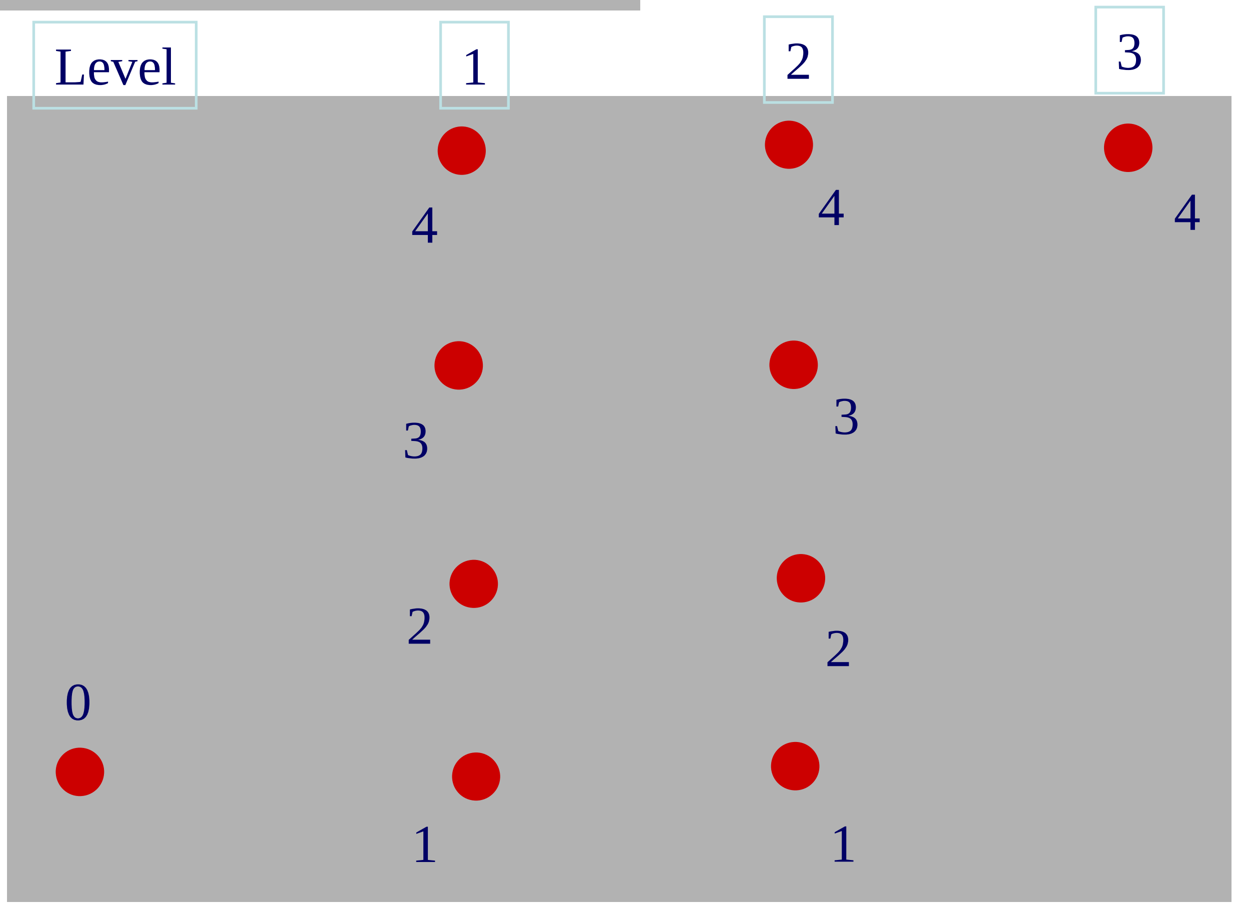
$$f_{ij}^{k} \in \{0,1\} \qquad i=0,\dots,n;\ j=1,\dots,n$$

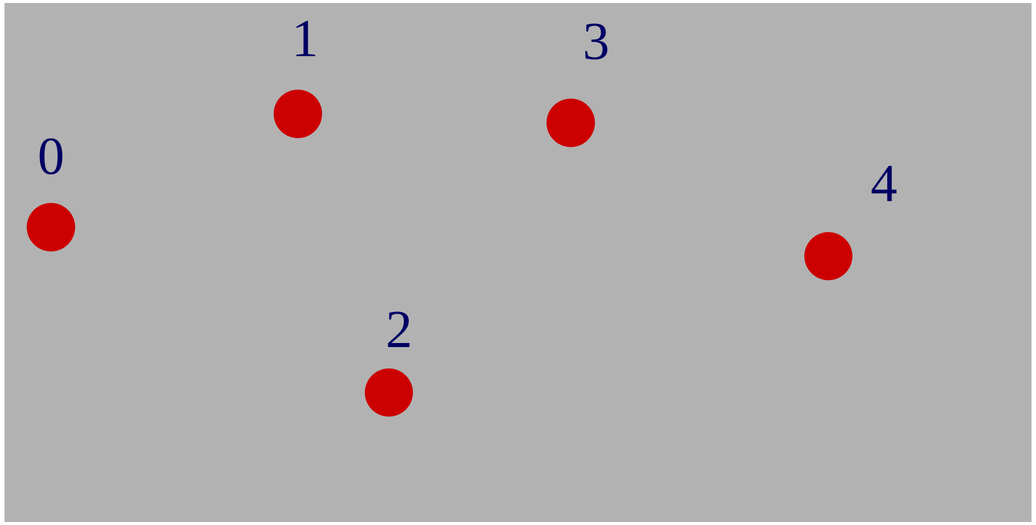
In general, F (HC-PATH(k)$_L$) contains fractional extreme points

“Exactly” modelled in
an adequate expanded graph (of polynomial size)



H=3





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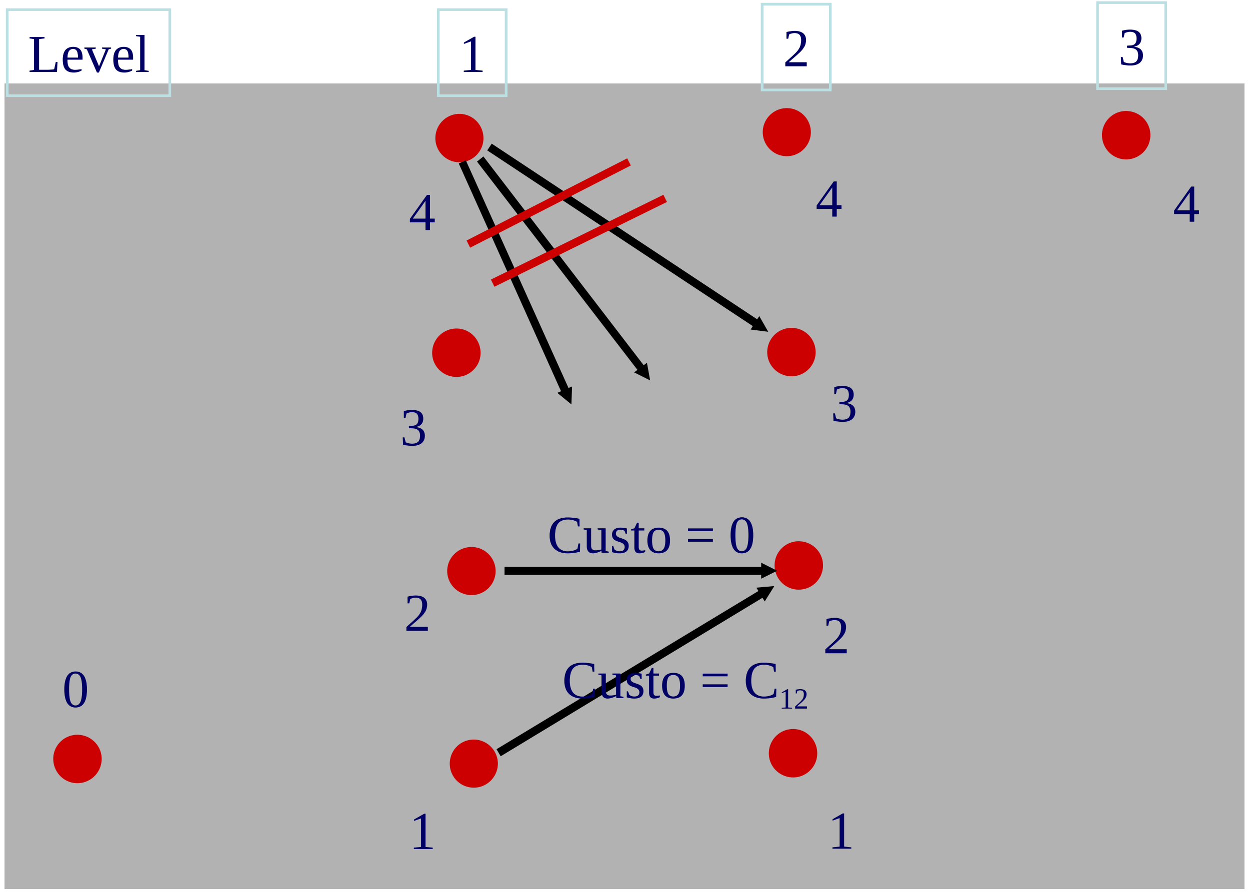

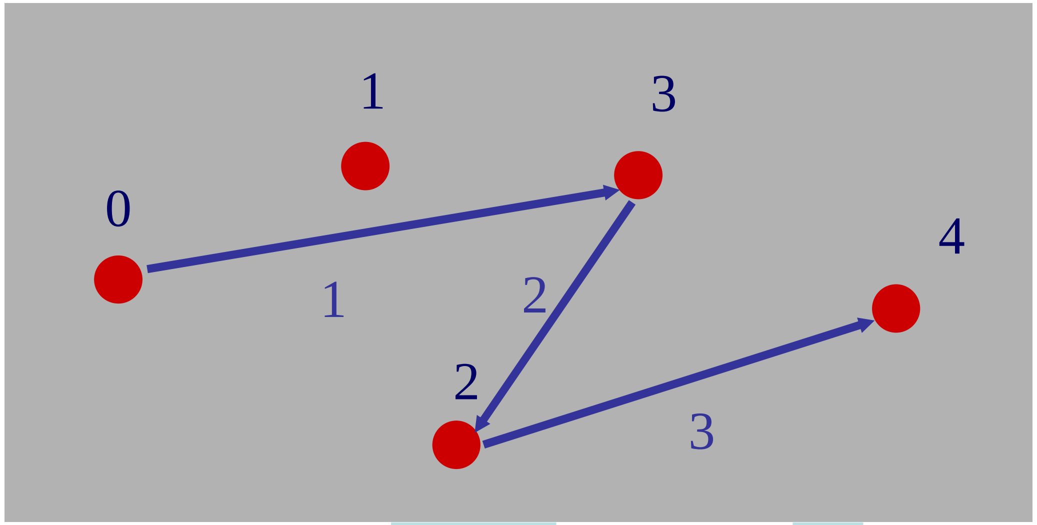

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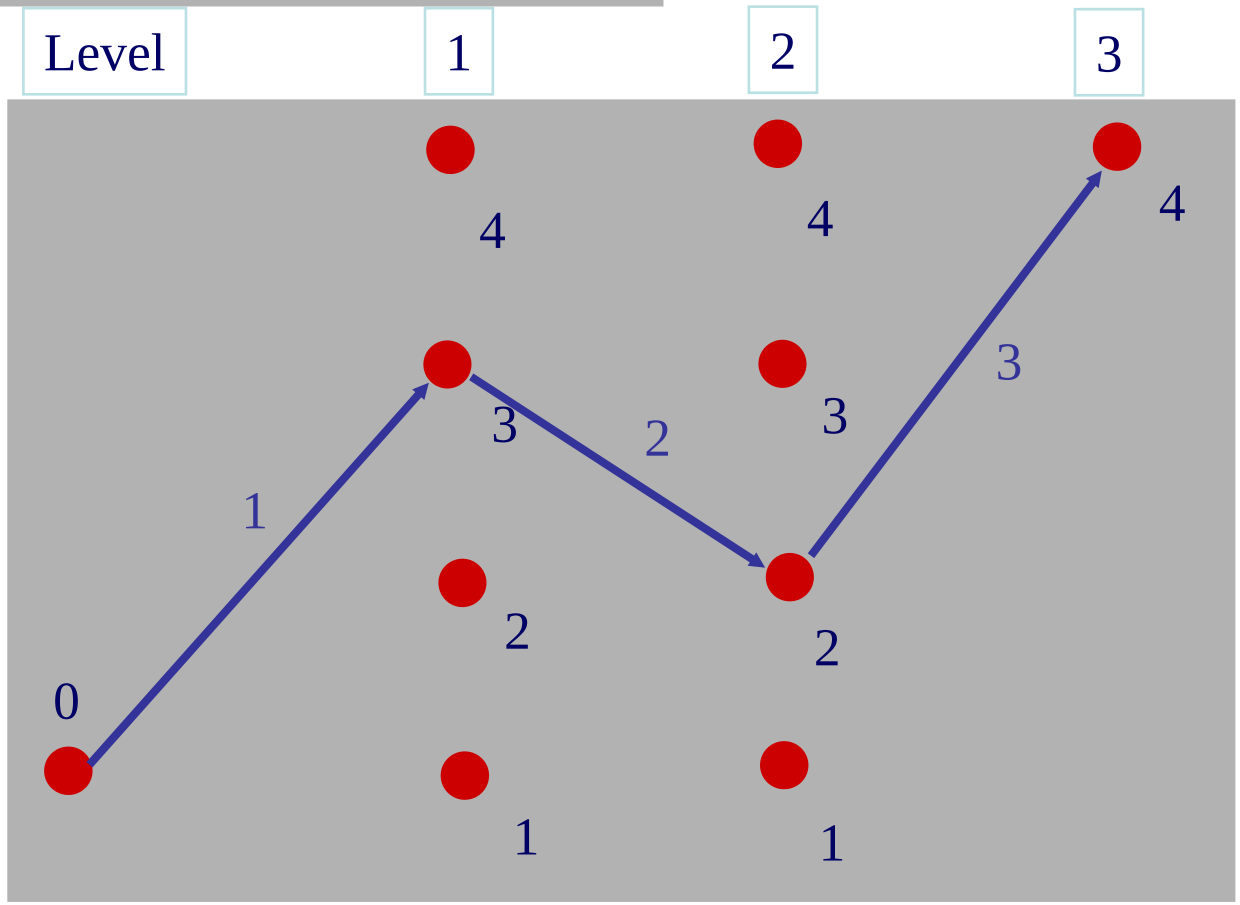

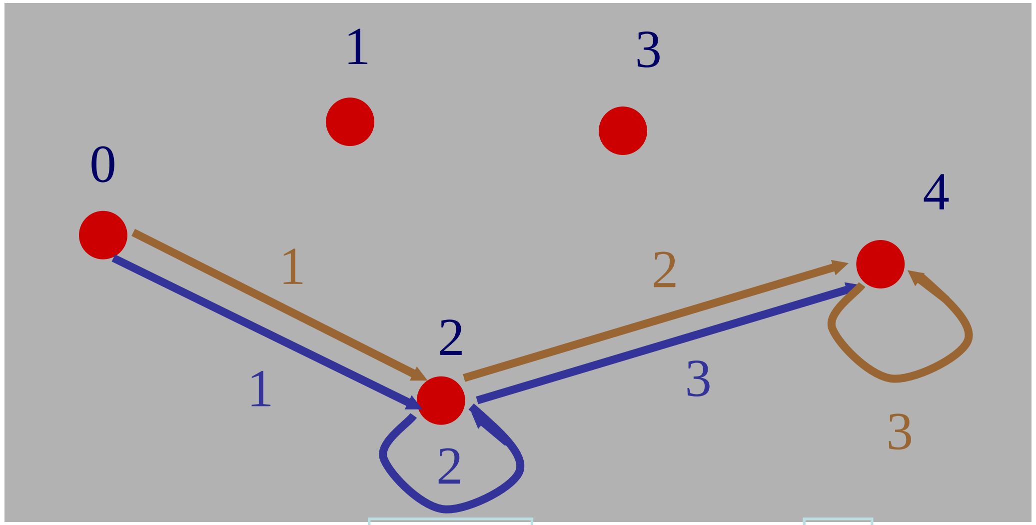

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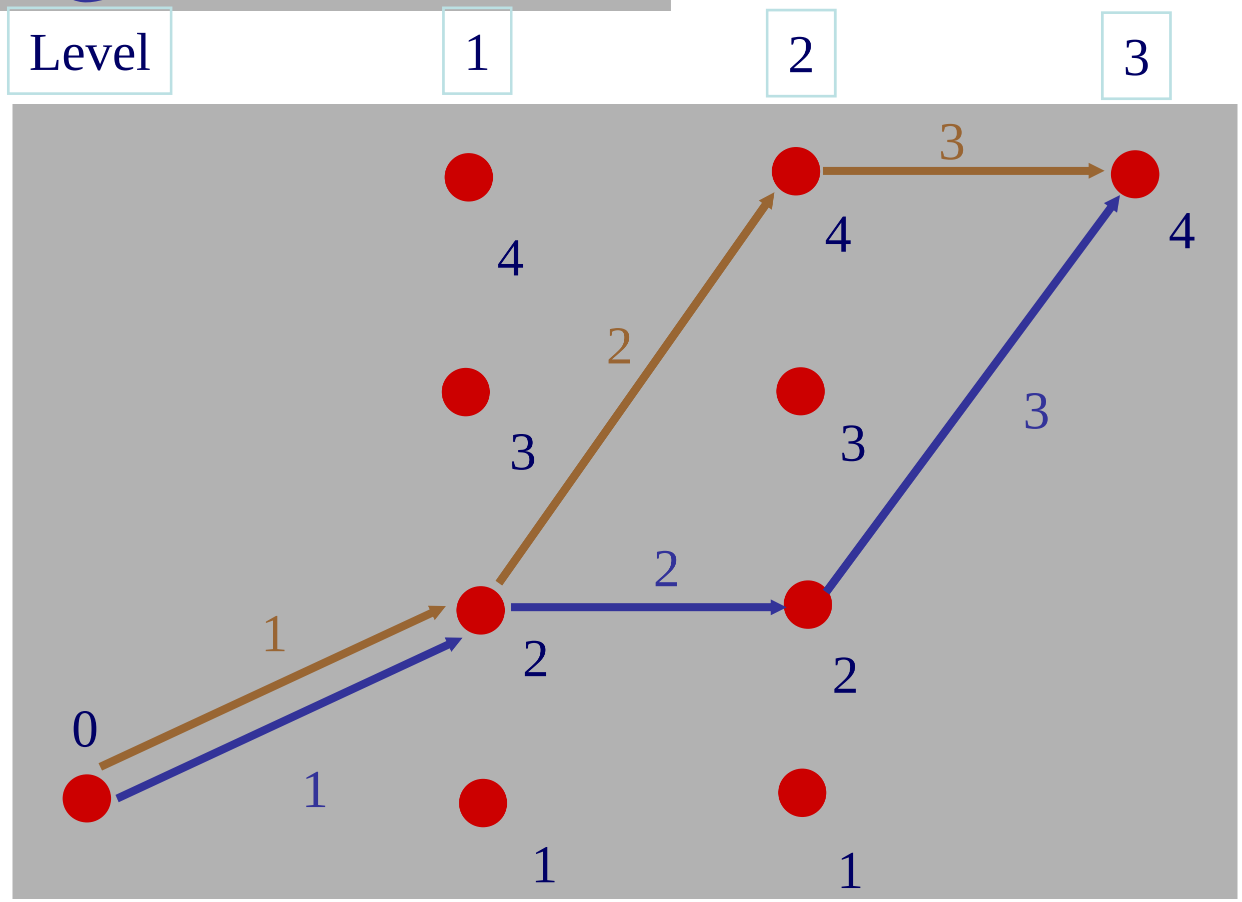

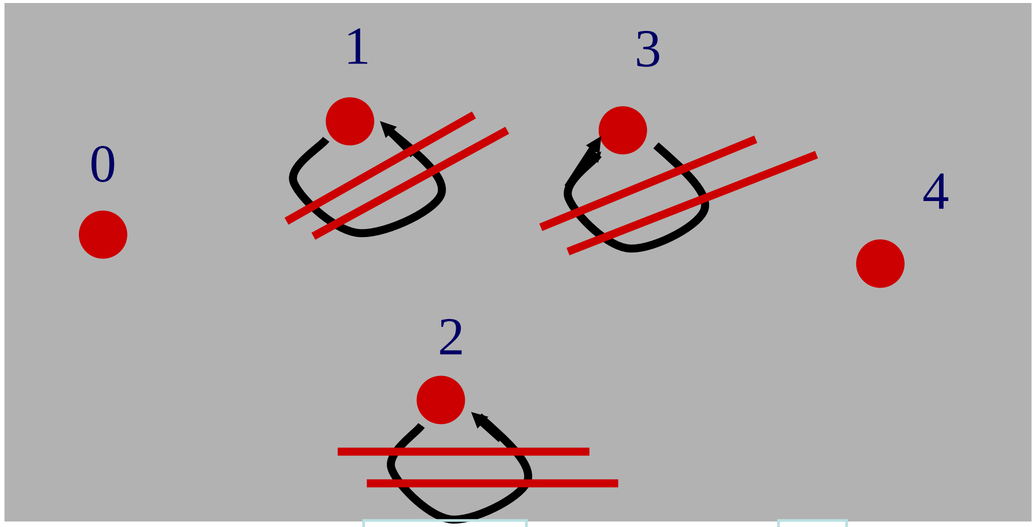

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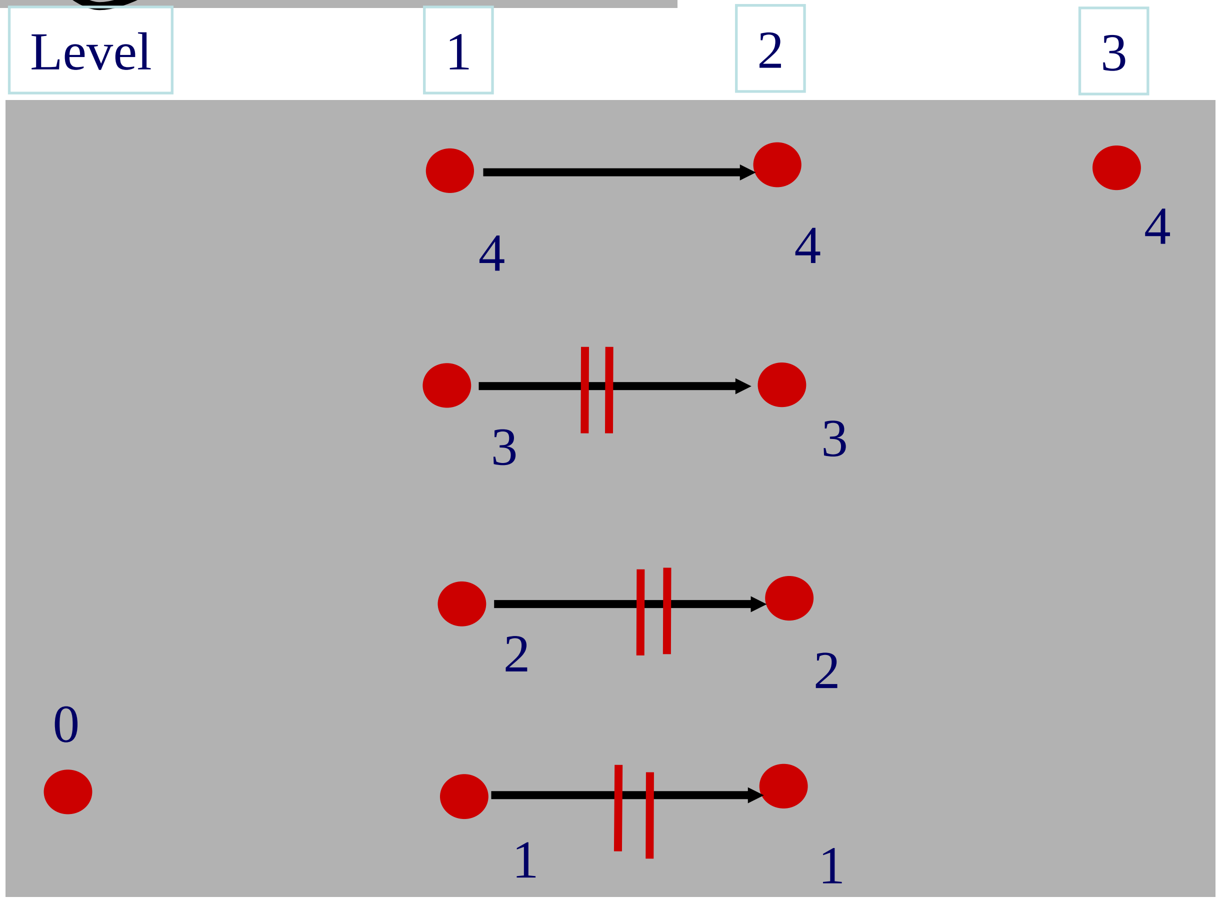

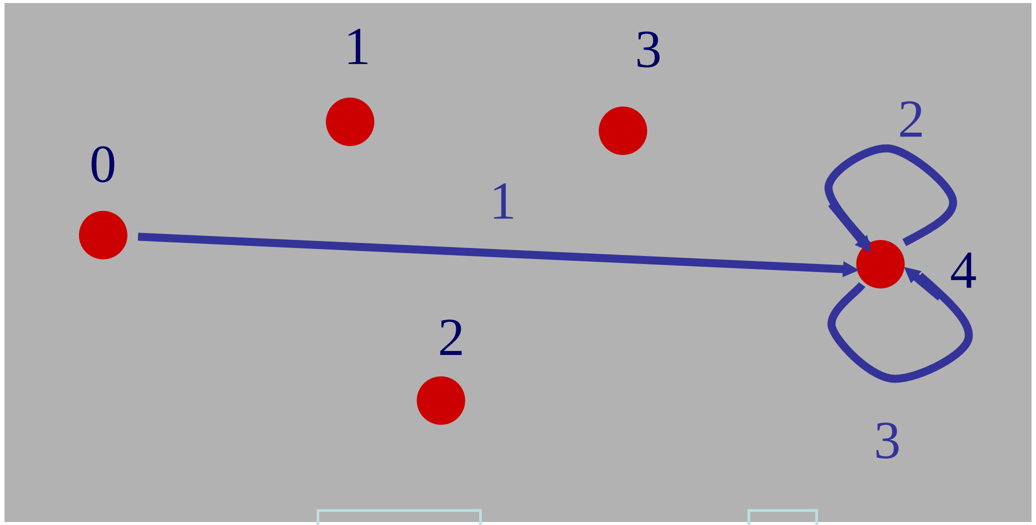

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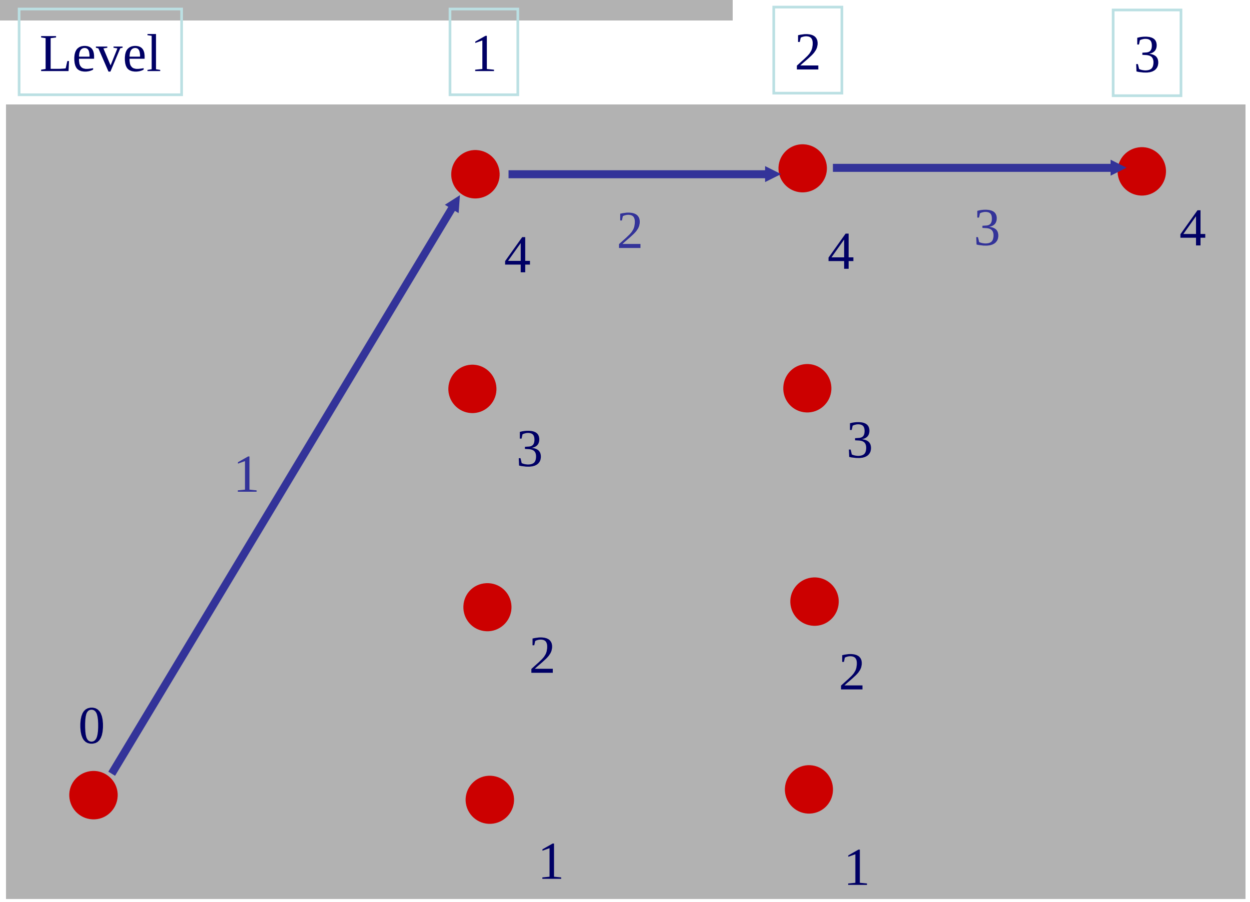
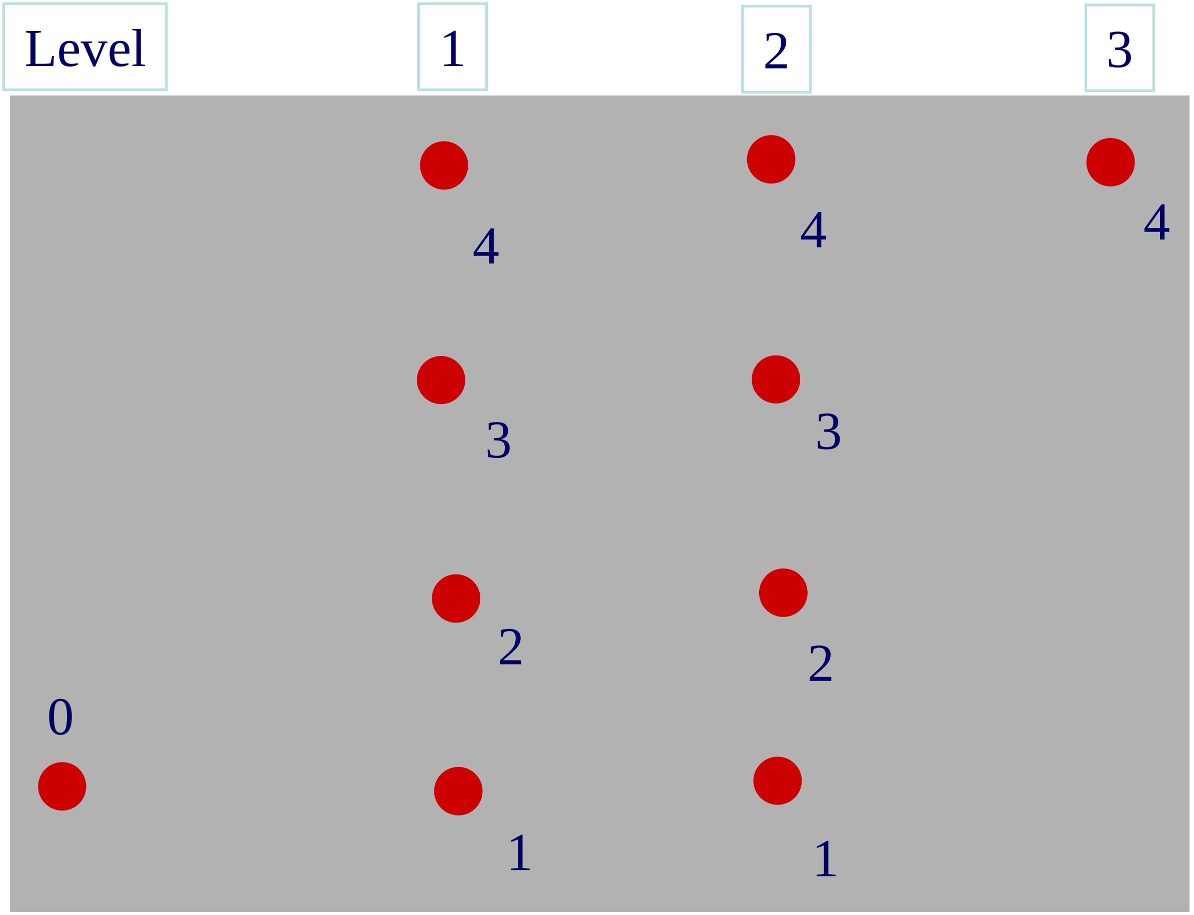


Expanded Graph

Unconstrained Path:
Usual Exact Formulation in Expanded Graph
Expanded Graph of Polynomial Size

Original Graph

Hop-Constrained Path
Extended Exact Formulation in Original Graph
Compact (of polynomial size) Formulation



Expanded Graph $z_{i^{h-1} j^{h}} \in \{0,1\}$ indicating whether arc (i^{h-1}, j^{h}) is in the solution

Original Graph $z_{ij}^{h} \in \{0,1\}$ indicating whether arc (i,j) is in position h in the solution

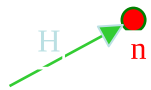
EXACT-HC-Path(k)

$$\min \sum_{i=0,\dots,n} \sum_{j=1,\dots,n} \sum_{h=1,\dots,H} d_{ij} Z_{ij}^{hk}$$

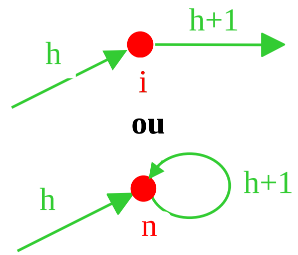
$d_{ij} = C_{ij}$ $\quad i \neq j$

$d_{nn} = 0$

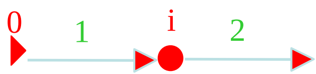
$$s.a \quad \sum_{j=1,\dots,n} Z_{jk}^{Hk} = 1$$



$$\sum_{j=1,\dots,n} Z_{ji}^{hk} = \sum_{j=1,\dots,n} Z_{ij}^{h+1,k} \qquad i = 1,\dots,n;$$



$$Z_{0i}^{1k} = \sum_{j=1,\dots,n} Z_{ij}^{2k} \qquad i = 1,\dots,n$$



$$Z_{ij}^{qk} \in \{0,1\} \qquad i = 0,\dots,n;\ j = 1,\dots,n, q = 1,\dots,H$$

$$\sum_{q=1,\dots H} Z_{ij}^{qk} = Y_{ij}^{k} \qquad i = 0,\dots,n;\ j = 1,\dots,n$$

MCF – generic with HC-Path(k) for each k

HDMCF - generic with Exact-HC-Path(k) for each k

Root	n	m	H	v(MCF_L)	v($HDMCF_L$)	OPT
TC	60	210	4	776.5	787.3	788
				(3868)	(21)	(5)
TE	60	210	4	1329.9	1404.5	1407
				(9962)	(125)	(110)