



The Commentariolus of Copernicus

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The Commentariolus of Copernicus

Some years before COPERNICUS consented to the publication of his large work *De Revolutionibus Orbium Caelestium* (1), he wrote a brief sketch (Commentariolus) of his astronomical system. The Commentariolus was not printed; a number of handwritten copies circulated for a time among students of the science, and then disappeared from view for three centuries. A copy found in Vienna was published by MAXIMILIAN CURTZE in 1878 (2); this Vienna MS will be referred to as V. A second copy found in Stockholm was published in 1881 (3); this will be referred to as S. On CURTZE's collation (4) of these two MSS LEOPOLD PROWE based the text (5) from which the present translation was made. It has been necessary to depart from PROWE's text in a few instances; these have been indicated in the notes. A third MS (6) is believed to exist in Leningrad; so far as I know, it has never been published.

A translation of the first section of the Commentariolus appears in PROWE (7). BIRKENMAJER published a partial translation of the work into Polish (8). A complete translation by ADOLF MÜLLER (*Zeitschrift für die Geschichte und Alterthumskunde Ermlands*, Braunsberg, 1899, vol. 12, pp. 359-82) came to my notice too late for use in this paper.

(1) All references in this paper to the *De Revolutionibus* are to the *Säkular-Ausgabe* (Thorn, 1873) for which the abbreviation is *Th.* The page-number is followed by a period and the line-number.

(2) *Mittheilungen des Copernicus-Vereins für Wissenschaft und Kunst* zu Thorn, I, 1878, 1-17.

(3) By ARVID LINDHAGEN in *Bihang till K. Svenska Vet. Akad. Handlingar*, Band 6, No. 12, Stockholm, 1881.

(4) *Mittheilungen des Copernicus-Vereins*, IV, 1882, 3-9.

(5) LEOPOLD PROWE "NICOLAUS COPERNICUS," 1883-4, vol. II, 184-202.

(6) LUDWIK A. BIRKENMAJER "MIKOLAJ KOPERNIK Wybór Pism" (*Biblioteka Narodowa*, Nr. 15, Serja I), Kraków 1926, XXVIII, fn. 28.

(7) *Op. cit.*, vol. I², 288-292.

(8) *Op. cit.*, 3-15.

NICHOLAS COPERNICUS

SKETCH OF HIS HYPOTHESES FOR THE HEAVENLY MOTIONS (9)

Our ancestors assumed, I observe, a large number of celestial spheres for this reason especially, to explain the apparent motion of the planets by the principle of regularity. For they thought it absurd that a heavenly body, which is a perfect sphere, should not always move uniformly. They saw that by connecting and combining regular motions in various ways they could make a heavenly body appear to be in any position.

CALLIPPUS and EUDOXUS, who endeavored to solve the problem by the use of concentric spheres, were unable to account for all the planetary movements; they had to explain not merely the apparent revolutions of the planets, but also the fact that these bodies appear to us sometimes to mount higher in the heavens, sometimes to descend; and this fact is incompatible with the principle of concentricity. Therefore it seemed better to employ eccentrics and epicycles, a system which most scholars finally accepted.

(9) PROWE writes "Die Formulierung der Ueberschrift... kann nicht von COPERNICUS selbst herrühren. Derselbe würde seine Darstellung des Weltgebäudes keineswegs als eine blosser *Hypothese* bezeichnet haben" (vol. II, 185, fn). But this objection rests on a misunderstanding of the COPERNICUS-OSIANDER controversy. COPERNICUS rejected OSIANDER's view that astronomical hypotheses are nothing but means of calculation: *Neque enim necesse est, eas hypotheses esse veras, imo ne verisimiles quidem, sed sufficit hoc unum, si calculum observationibus congruentem exhibeant...* (Th 1.14-16). But to reject this view of hypotheses is not to reject hypotheses. For COPERNICUS a hypothesis is not only a means of calculation but also a statement of what is physically true. Thus the real motion of the earth is a hypothesis: *Id enim ex hypothesi motus terrae sequi videtur...* (Th 163.2); ... *quae omnia huic quoque nostrae hypothesi mobilitatis terrae... plane sunt convenientia* (Th 345.20-21); ... *per hanc hypothesim mobilitatis terrae...* (Th 357.12); ... *nostrae hypothesi mobilitatis terrae...* (Th 365.5-6). DREYER, too, went astray on this point, for he says "... to COPERNICUS the motion of the earth was a physical reality and not a mere working hypothesis. Not to speak of the fact that he nowhere in his work calls it a hypothesis..." (J. L. E. DREYER, *History of the Planetary Systems*, 320). Note also ... *inter principia et hypotheses assumpserimus non errantium stellarum sphaeram omnino immobilem esse...* (Th 109.4-5). COPERNICUS speaks of his lunar theory as *nostras de luna hypotheses* (Th 275.6-7), and remarks... *quae admodum congruunt observationi, quo minus dubitaverit aliquis nostras hypotheses, et quae ex eis prodita sunt, recte se habere* (Th 298.7-9).

Yet the planetary theories of PTOLEMY and most other astronomers, although consistent with the numerical data, seemed likewise to present no small difficulty. For these theories were not adequate unless certain equants were also conceived; it then appeared that a planet moved with uniform velocity neither on its deferent nor about the center of its epicycle. Hence a system of this sort seemed neither sufficiently absolute nor sufficiently pleasing to the mind.

Having become aware of these defects, I often considered whether there could perhaps be found a more reasonable arrangement of circles, from which every apparent inequality would be derived and in which everything would move uniformly, as a system of absolute motion requires. After I had addressed myself to this very difficult and almost insoluble problem, the suggestion at length came to me how it could be solved with fewer and much simpler constructions than were formerly used, if some assumptions (which are called axioms) were granted me. They follow in this order.

ASSUMPTIONS (10)

1. There is no one center of all the celestial circles or spheres (11).
2. The center of the earth is not the center of the universe, but only of gravity and of the moon's orbit.
3. All the planets revolve about the sun as their mid-point, and therefore the sun is the center of the universe.
4. The ratio of the earth's distance from the sun to the height of the firmament is so much smaller than the ratio of the earth's radius to its distance from the sun that the distance from the earth to the sun is imperceptible in comparison with the height of the firmament.

(10) In his description of the *Commentariolus* DREYER incorrectly states the number of assumptions as six (*op. cit.*, 317). The source of his mistake is probably the oversight in PROWE, vol. I², 291, which PROWE himself calls attention to and corrects (vol. II, 187 fn).

(11) There is a grave error in PROWE's translation "Für alle Himmels-Körper und deren Bahnen giebt es nur *einen* Mittelpunkt" (vol. I², 290). COPERNICUS denies (not asserts) that there is only one central point; cf. *Pluribus ergo existentibus centris...* (*Th* 24.23).

5. Whatever motion appears in the firmament arises not from any motion of the firmament, but from the earth's motion. The earth together with its circumjacent elements (12) performs a complete rotation on its poles in a daily motion, while the unmoved firmament and highest heaven abide unchanged.

6. What appear to us as motions of the sun arise not from its motion but from the motion of the earth and our sphere, with which we revolve about the sun like any other planet. The earth has, then, more than one motion.

7. The apparent retrograde and direct motion of the planets arises not from their motion but from the earth's. The motion of the earth alone, therefore, suffices to explain so many apparent inequalities in the heavens.

Having set forth these assumptions, I shall endeavor briefly to show how uniformity of the motions can be saved in a systematic way. However, I have thought it well, for the sake of brevity, to omit from this sketch mathematical demonstrations, reserving these for my larger work. But in the explanation of the circles I shall set down here the lengths of the radii; and from these the reader who is not unacquainted with mathematics will readily perceive how closely this arrangement of circles agrees with the numerical data and observations.

Accordingly, let no one suppose that I have gratuitously asserted, with the Pythagoreans, the motion of the earth; strong proof will be found in my exposition of the circles. For the principal arguments by which the natural philosophers attempt to establish the immobility of the earth rest for the most part on the appearances; it is particularly such arguments that collapse here, since I explain the appearances also.

THE ORDER OF THE SPHERES

The celestial spheres are arranged in the following order. The highest is the immovable sphere of the fixed stars, which contains

(12) These are (a) the atmosphere and (b) the waters that lie upon the surface of the earth. *Vide infra*, in the discussion of the earth's rotation, *Sic quidem terra cum circumfluis aqua et vicino aere volvitur*; cf. *De Rev.* I, 8., *non solum terra cum aqueo elemento sibi coniuncto sic moveatur, sed non modica quoque pars aeris, et quaecumque eodem modo terrae cognationem habent* (*Th* 22.15-17).

and gives position to all things. Beneath it is Saturn, which Jupiter follows, then Mars. Below Mars is the sphere on which we revolve; then Venus; last is Mercury. The moon revolves about the center of the earth, and moves with the earth like an epicycle. In the same order also, one planet surpasses another in speed of revolution, according as they trace greater or smaller circles. Thus Saturn completes its revolution in thirty years, Jupiter in twelve, Mars (13) and the earth in one; Venus in nine months, Mercury in three.

THE APPARENT MOTIONS OF THE SUN

The earth has three motions. First, it revolves annually in a great circle about the sun in the order of the signs, always describing equal arcs in equal times; the distance from the center of the circle to the center of the sun is $1/25$ of the radius of the circle (14). The radius is assumed to have a length imperceptible in comparison with the height of the firmament; consequently the sun appears to revolve with this motion, as if the earth lay in the center of the universe. However, this appearance is caused by the motion not of the sun but of the earth, so that for example when the earth is in the sign of Capricorn, the sun is seen diametrically opposite in Cancer, and so on. On account of the previously mentioned distance of the sun from the center of the circle, this apparent motion of the sun is not uniform, the maximum inequality being $2 \frac{1}{6}^{\circ}$. The line drawn from the sun through the center of the circle is invariably directed toward a point of the firmament about 10° west of the more brilliant (15) of the two

(13) This is the reading of both V and S. But below in the section on the superior planets Mars' period is given as 29 months; and in *De Rev.* I, 10 as two years: *Deinde Mars, qui biennio circuit* (*Th* 29.6); the explanatory figure there likewise has *Martis bima revolutio*. In his edition of V CURTZE inserted [*tertio*] after *Mars*, and PROWE dropped the square brackets. BIRKENMAJER's translation runs (9) "Mars revolves in not quite two years."

(14) In *De Rev.* COPERNICUS regards this eccentricity as varying (*Th* 209.27-210.1) between a maximum of $1/24$ and a minimum of $1/31$ (*Th* 219.31-220.26). PTOLEMY's observations make the eccentricity $0.0414 = 1/24$ (*Th* 209.12-13); COPERNICUS' observations, $0.0323 = 1/31$ (*Th* 211.18-19, 23-25).

(15) Pollux, β Geminorum; cf. COPERNICUS' star catalogue *Th* 132.31-32. Since the longitude of Pollux is there given as $79^{\circ} 50'$, the longitude of the point would

bright stars in the head of Gemini. Therefore when the earth is opposite this point, and the center of the circle lies between them, the sun is seen at its greatest distance from the earth. In this circle, then, the earth revolves together with whatever else is included within the moon's orbit.

The second motion, which is peculiar to the earth, is the daily rotation on the poles in the order of the signs, that is, from west to east. On account of this rotation the entire universe appears to revolve with enormous speed. Thus does the earth rotate together with its circumjacent waters and encircling atmosphere.

The third is the motion in declination. For the axis of the daily rotation is not parallel to the axis of the great circle but is inclined at an angle that varies with the position on the circumference; at present the angle is about $23\frac{1}{2}^{\circ}$ (16). Therefore, while the center of the earth always remains in the plane of the ecliptic, that is, in the circumference of the great circle, the poles of the earth rotate, both of them describing small circles about centers equidistant from the axis of the great circle. The period of this motion is not quite a year and is nearly equal to the annual revolution on the great circle. But the axis of the great circle is invariably directed toward the points of the firmament which are called the poles of the ecliptic. In like manner the motion in declination, combined with the annual motion in their joint effect upon the poles of the daily rotation, would keep these poles constantly fixed at the same points of the heavens, if the periods of both motions were exactly equal (17). Now with the long passage of time it has become clear that this inclination

be $69^{\circ} 50'$. As BIRKENMAJER points out (*op. cit.*, 10, fn. 18), in *De Rev.* COPERNICUS abandons this doctrine of the fixity of the solar apse: *Instat iam maior difficultas circa absidis solaris inconstantiam... quam Ptolemaeus ratus est esse fixam...* (*Th* 216.3-4).

(16) In *De Rev.* II, 2 COPERNICUS states that he and certain of his contemporaries have found this angle (which is equal to the obliquity of the ecliptic) to be not greater than $23^{\circ} 29'$ (*Th* 76.29-77.1); and in III, 2 he says *nostris autem temporibus non invenitur maior partibus XXIII, scrupulis XXVIII s.* (*Th* 162.24-25). NEWCOMB's determination of the obliquity for 1900 was $23^{\circ} 27' 8''.26$; on the basis of an annual diminution of $0''.4684$, the value for 1540 would be $23^{\circ} 29' 57''$.

(17) This obviously requires the direction of the motion in declination to be opposite to the direction of the annual motion. The explicit statement appears in *De Rev.* I, 11 *Sequitur ergo tertius declinationis motus annua quoque revolutione, sed in praecedentia, hoc est contra motum centri reflectens* (*Th* 31.22-24).

of the earth to the firmament changes; hence it has seemed to most persons that the firmament has several motions in conformity with a law not yet sufficiently understood. But it would be less surprising if all these changes should occur on account of the motion of the earth. I am not concerned to state what the path of the poles is. I am aware that, in lesser matters, a magnetized iron needle always points in the same direction. It has nevertheless seemed a better view that the path is a circle by the motion of which the motion of the poles is governed. This circle must doubtless be sublunar.

EQUAL MOTION SHOULD BE MEASURED NOT BY THE EQUINOXES
BUT BY THE FIXED STARS

Since the equinoxes and the other cardinal points of the universe shift considerably, whoever attempts to derive from them the equal length of the annual revolution necessarily falls into error. Different determinations of this length were made in different ages on the basis of many observations. HIPPARCHUS computes it as $365\frac{1}{4}$ days (18); ALBATEGNIUS the Chaldean as $365^d 5^h 46^m$ (19), that is, 13^m and $3/5^m$ or $1/3^m$ less than PTOLEMY (20). HISPALENSIS (21) increases ALBATEGNIUS' estimate by the 20th part

(18) This incorrect statement about HIPPARCHUS is changed in *De Rev.* There he is correctly said to have declared that the year was less than $365 \frac{1}{4}$ days (*Th* 191.31-192.2) by $1/300$ th of a day (*Th* 237.13-15); and it is to CALLIPPUS, ARISTARCHUS, and ARCHIMEDES that the value $365 \frac{1}{4}$ days is assigned (*Th* 191.26-28). HIPPARCHUS' estimate, $365 \frac{1}{4}$ days — $1/300$ day = $365^d 5^h 55^m 12^s$, is quoted in *Almagest* III, 1 (ed. HEIBERG, I, 207.24-208.1).

(19) This agrees with the statement in *De Rev.* III, 13 where ALBATEGNIUS' estimate is correctly given as $365^d 5^h 46^m 24^s$ (*Th* 193.7-8); cf. C. A. NALLINO "Al-Battānī sive Albatenii Opus Astronomicum" (*Pubblicazioni del Reale Osservatorio di Brera* in Milano, 1903, No. 40, Part I, 42).

(20) The value incorrectly assigned here to PTOLEMY is just short of $365 \frac{1}{4}$ days. But in *De Rev.* III, 13 the value $365^d 5^h 55^m 12^s$ is correctly assigned to him (*Th* 192.2-3, 21-23), for PTOLEMY accepted HIPPARCHUS' estimate (*Almagest* III, 1, ed. HEIBERG, I, 208.3-14). Observe that *scrupulum primum* = 24 minutes, *scrupulum secundum* = 24 seconds.

(21) PROWE (vol. II, 191, fn) followed CURTZE (*Mittheilungen des Copernicus-Vereins*, I, 1878, 10) in supposing that HISPALENSIS, i.e. from Hispalis = Seville, here means ISIDORE of Seville. In COPERNICUS' view precession attained its greatest rapidity in the time of ALBATEGNIUS; thereafter diminution set in: *E quibus patet, tardiores fuisse praecessiones aequinoctiorum ante Ptolemaeum in illis CCC annis quam a Ptolemaeo ad Albategnium, et hanc quoque velociorem ab Albategnio ad nostra tempora* (*Th* 162.14-17). Therefore the shortest length

of an hour, since he determines the tropical year as $365^d 5^h 49^m$.

Lest these differences should seem to have arisen from errors of observation, let me say that if any one will study the details carefully, he will find that the discrepancy has always corresponded to the motion of the equinoxes. For when the cardinal points moved 1° in 100 years, as they were found to be moving in the age of PTOLEMY, the length of the year was then what PTOLEMY stated it to be. When however in the following centuries they moved with greater rapidity, being opposed to lesser motions, the year became shorter; and this decrease corresponded to the increase in precession. For the annual motion was completed in a shorter time on account of the more rapid recurrence of the equinoxes. Therefore the derivation of the equal length of the year from the fixed stars is more accurate. I used Spica

of the tropical year fell in the time of ALBATEGNIUS; and the increase noted by HISPALENSIS must be associated with a later astronomer. This chronological consideration rules out ISIDORE immediately. Moreover, an examination of the astronomical portions of his extant works (J. P. MIGNÉ, *Patrologia Latina*, vols. 81-84) shows that he gives 365 days as the length of both the tropical and sidereal years.

Who, then, is HISPALENSIS? JĀBIR IBN AFLĀḤ? In 1534 PETER APIAN'S *Instrumentum Primi Mobilis* was published together with GEBRI FILII AFFLA HISPALENSIS... *Libri IX de Astronomia*. A copy was given by RHETICUS to COPERNICUS (*Mittheilungen des Copernicus-Vereins*, I, 1878, 36), and hence it did not get into his hands until 1539 (*Th* 447.5-7, 490.37-38). But all our evidence points to 1533 as the *terminus ante quem* the Commentariolus was written. Moreover, JĀBIR (*op. cit.*, 38-39) simply repeats the HIPPARCHUS-PTOLEMY estimate of the length of the tropical year. Clearly he is not the HISPALENSIS to whom COPERNICUS refers.

BIRKENMAJER proposed ALFONSO of Cordova (MIKOLAJ KOPERNIK *Wybór Pism*, 12, fn. 24). In the Appendix (1555) to CONRAD GESNER'S *Bibliotheca Universalis* the following entry occurs: ALPHONSUS *Hispalensis de Corduba, artium et medicinae doctor, tabulas Astronomicas ac in easdem demonstrationum theorematum, Elizabethae reginae dicavit. Impressae sunt Venetijs cum tabulis Alphonsi regis. Claruit anno D. 1484*. Now COPERNICUS owned a copy of the 1492 edition of the Alfonsine Tables (PROWE, vol. I², 417). It may well be, then, that the HISPALENSIS of our text is ALFONSO of Cordova. The identification must await an examination of his works, at present inaccessible to the translator (cf. *Gesamtkatalog der Wiegendrucke*, II, 1573; *Catalogue Général des Livres Imprimés de la Bibliothèque Nationale*, 1924, II, 558, 562; RICO Y SINOBAS "Libros del Saber de Astronomia," V, 78, 90, 139-144; and perhaps *Catalogue of Books Printed in the XVth Century now in the British Museum*, V, 424, IA23354).

Meanwhile, I may suggest that COPERNICUS is perhaps citing a translation to which the name of JOHN of Seville (JOANNES HISPALENSIS) was attached. I am at present unable definitely to substantiate or reject this suggestion.

Virginis and found that the year has always been 365 days, 6 hours, and about 10 minutes (22), which is also the estimate of the ancient Egyptians. The same method must be employed also with the other motions of the planets, because their apsides and stationary points reveal the laws of celestial motion and heaven itself with true testimony.

THE MOON

The moon seems to me to have four motions in addition to the annual revolution which has been mentioned. For it revolves once a month on its deferent circle about the center of the earth in the order of the signs. The deferent carries the epicycle which is commonly called the epicycle of the first inequality or argument, but which I call the first or greater epicycle (23). In the upper portion of its circumference this greater epicycle revolves in the direction opposite to that of the deferent (24), and its period is a little more than a month. Attached to it is a second epicycle. The moon, finally, moving with this second epicycle, completes two revolutions a month in the direction opposite to that of the greater epicycle, so that whenever the center of the greater epicycle crosses the line drawn from the center of the great circle through the center of the earth (I call this line the diameter of the great circle), the moon is nearest to the center of the greater epicycle. This occurs at new and full moon; but contrariwise at the mid-points, which are the quadratures, the moon is most remote from the center of the greater epicycle. The length of the radius (25) of the greater

(22) COPERNICUS' estimate of the length of the sidereal year is stated more exactly in *De Rev.* III, 14 as $365^d 6^h 9^m 40^s$ (*Th* 195.29-196.3); PROWE'S $365^d 6^h 8^m 40^s$ is a slip (vol. II, 191, last paragraph of fn). NEWCOMB'S determination (1900) is $365^d 6^h 9^m 9^s.54$.

(23) The meaning of *anni* is not clear to the translator.

(24) When the motion of a circle, in the upper portion of its circumference, is in precedence i.e. from east to west, in the lower portion it is in consequence, from west to east; and vice versa. "... *motus epicycli intelligatur ex c in b, et b in a, hoc est superne in praecedentia, inferne ad consequentia...*" (*Th* 251.27-252.1). When the direction of a motion is stated without reference to the portion of the circumference, it is the upper circumference that is understood.

(25) The text here rests on S alone, because there is a lacuna in V. *diametri*, the reading of S, is clearly wrong and must be changed to *semidiametri*. PROWE

epicycle is to the radius of the deferent as $1 \frac{1}{18} : 10$; and to the radius of the smaller epicycle as $4 \frac{3}{4} : 1$ (26).

Thus the moon appears at times rapidly, at times slowly to descend and ascend; and the motion of the smaller epicycle adds two irregularities to the first inequality. For it withdraws the moon from uniform motion on the circumference of the greater epicycle, the maximum inequality being $17 \frac{1}{4}^{\circ}$; and it brings the center of the greater epicycle at times nearer the moon, at times further from it, within the limits of the radius of the smaller epicycle (27). Therefore since the moon describes unequal circles about the center of the greater epicycle, the first inequality varies considerably. In conjunctions and oppositions to the sun its greatest value does not exceed $4^{\circ} 56'$, but in quadratures it increases to $6^{\circ} 36'$ (28). Those who account for this variation by an eccentric circle, in addition to having an unsatisfactory inequality of motion on the eccentric (29), fall into two manifest errors. For the consequence by mathematical analysis is that the moon in quadrature, when it is in the lowest part of the epicycle, should appear nearly four times greater (if the entire disk were luminous) than when new and full, unless its magnitude increases and diminishes in no reasonable way. So too, because the size of the earth is sensible in comparison with its distance from the moon, the lunar parallax should increase very greatly at the qua-

overlooked this, even though he cited certain of COPERNICUS' "errechneten Werthe für den Semidiameter der Epicykel der Planeten" to support another emendation in the same sentence (vol. II, 193, fn).

(26) The first ratio $1 \frac{1}{18} : 10 = 1055 : 10,000$ is altered in *De Rev.* IV, 8 to $1097 : 10,000 = 1 \frac{1}{10} : 10$. The second ratio appears there as $1097 : 237 = 4.63 : 1$ (*Th* 258.7-11).

(27) Uncertainties of text and difficulties of syntax make the translation of this sentence hazardous. *De circumferentia ipsa quantitatis seu diametri respondentis eum* has been omitted from the translation.

(28) In *De Rev.* IV, 11 the greatest value for conjunctions and oppositions is likewise given as $4^{\circ} 56'$; but the greatest additional value for the inequality in quadratures is there stated to be $2^{\circ} 44'$ (*Th* 262.23-32). The sum $7^{\circ} 40'$ is the traditional Ptolemaic value, which COPERNICUS accepted. His tables give the sum of the two inequalities for 90° and 270° , the quadratures, as $7^{\circ} 35'$ (*Th* 264.35). I suggest therefore that the figure in our text should be changed from $6^{\circ} 36'$ to $7^{\circ} 36'$. Again the reading of S cannot be checked on account of the lacuna in V.

(29) *Dum enim fatentur motum centri epicycli aequalem esse circa centrum terrae, fateri etiam oportet inaequalem esse in orbe proprio, quem describit, eccentro* (*Th* 233.11-13).

dratures. But careful investigation will show that in both respects the quadratures differ very little from new and full moon. Accordingly it will be readily admitted that my explanation is the truer. With these three motions in longitude, then, the moon passes through the points of its motion in latitude. The axes of the epicycles are parallel to the axis of the deferent, and therefore the moon does not move out of the plane of the deferent.

But the axis of the deferent is inclined to the axis of the great circle or ecliptic; hence the moon moves out of the plane of the ecliptic. Its inclination occurs within the limits of an angle which subtends 5° of the circumference of a circle. The poles of this circle revolve at an equal distance from the axis of the ecliptic, just as has been explained regarding declination. But in the present case they move in the reverse order of the signs and much more slowly, the period of the revolution being nineteen years. It seems to most astronomers that the motion takes place in the higher circle to which the poles are attached as they revolve in the manner described. Such a fabric of motions, then, does the moon seem to have.

THE THREE SUPERIOR PLANETS SATURN — JUPITER — MARS

Saturn, Jupiter, and Mars have a similar system of motions, since their deferents completely enclose the great circle and revolve in the order of the signs about its center as their common center. Saturn's deferent revolves in 30 years, Jupiter's in 12 years, and Mars' in 29 months (13); it is as though the size of the circles delayed the revolutions. For if the radius of the great circle is divided into 25 units, the radius of Mars' deferent will be 30 units (30), Jupiter's $130 \frac{1}{5}$, and

(30) The sizes of these circles are altered in *De Rev.* The radius (R) of the concentric deferent (Commentariolus) is equal to the radius (R) of the eccentric (*De Rev.*). Likewise, the radius (E) of the first epicycle (*Commentariolus*) is equal to the eccentricity (E) of the eccentric (*De Rev.*). Let r denote the radius of the great circle.

Then in *De Rev.* for Saturn $R = 10,000$; $r = 1090$ (*Th* 341.29); $E = 854$ (*Th* 330.18); for Jupiter $R = 10,000$; $r = 1916$ (*Th* 353.15-16); $E = 687$ (*Th* 348.17-25); for Mars $R = 10,000$; $r = 6580$ (*Th* 364.8-10); $E = 1460$

Saturn's $230 \frac{1}{6}$ (31). I define "radius of the deferent" as the distance from the center of the deferent to the center of the first epicycle. Each deferent has two epicycles (32), one of which carries the other, in much the same way as was explained in the case of the moon, but with a different arrangement. For the first epicycle revolves in the direction opposite to that of the deferent, the periods of both being equal. The second epicycle, carrying the planet, revolves in the direction opposite to that of the first with twice the velocity, so that whenever the second epicycle is at its greatest or least distance from the center of the deferent, the planet is nearest to the center of the first epicycle; and when the second epicycle is at the mid-points,

(*Th* 358.28-29); for Venus $R = 7193$ (*Th* 367.13-14); $r = 10,000$; E (mean value) = 263 (*Th* 368.30-369.3, 369.8-11); for Mercury R (mean value) = 3763; $r = 10,000$; E (mean value) = 736 (*Th* 381.2-4, 381.32-382.10, 382.27-383.2).

	R : r		R : E	
	<i>Commentariolus</i>	<i>De Rev.</i>	<i>Commentariolus</i>	<i>De Rev.</i>
Saturn	10,000 : 1086	10,000 : 1090	10,000 : 855	10,000 : 854
Jupiter	10,000 : 1920	10,000 : 1916	10,000 : 776	10,000 : 687
Mars	10,000 : 8333	10,000 : 6580	10,000 : 1856	10,000 : 1460
Venus	7200 : 10,000	7193 : 10,000	24 : 1	27 : 1
Mercury	3760 : 10,000	3763 : 10,000	5.6 : 1	5.1 : 1

(31) $V : 236 \frac{1}{6}$; $S : 230 \frac{1}{6}$. CURTZE's collation inaccurately assigns to Jupiter the reading of S for Saturn. PROWE accepted the reading of V , but S is to be preferred for the following reason. CURTZE discovered some astronomical notations made by COPERNICUS in his copy of the Tables of REGIOMONTANUS. CURTZE used one of these notations to emend a reading of S relating to the second epicycle of the moon, where V has a lacuna; and remarks "Die Notizen... scheinen also speciell für den Gebrauch bei der Abfassung des *Commentariolus* berechnet zu sein" (*Mitteilungen des Copernicus-Vereins*, IV, 1882, 7). But he overlooked the fact that a few lines above the entry concerning the moon there is an entry concerning Saturn which agrees with S and not V (PROWE, vol. II, 211).

(32) In *De Rev.* COPERNICUS replaces this arrangement of a concentric deferent and two epicycles by an eccentric deferent with a single epicycle. He points out the geometric equivalence of the two devices: *Quod si loco ab eccentrici caperemus ipsi aequalem in d homocentrum, qui deferat epicyclium, cuius quae ex centro fuerit aequalis ipsi dc, in hoc ipso quoque alterum epicyclium, cuius dimetiens sit dimidium ipsius cd; moveatur autem primus epicyclus in consequentia, secundus tantumdem in diversum, in quo demum planetes duplicato reflectatur motu: accident eadem, quae iam diximus...* (*Th* 327.6-12). The reason for the substitution is the variation in the eccentricity of the great circle: *Sed elegimus hic eccentrici epicyclum, eo quod manente semper inter solem et c centrum d interim mutasse reperitur, ut in solaribus apparentiis ostensum est* (*Th* 327.13-16). In the *Commentariolus* this eccentricity is regarded as constant; *vide supra* the section on The Apparent Motions of the Sun and note 14.

a quadrant's distance from the two points just mentioned, the planet is most remote from the center of the first epicycle. Through the combination of these motions of the deferent and epicycles, and by reason of the equality of their revolutions, the elongations and progressive motions of the planets follow unchanging paths in the firmament. Hence the planets preserve throughout their motion fixed patterns, and likewise have invariable apsides (33); Saturn, near the star which is said to be above the elbow of Sagittarius (34); Jupiter, 8° behind the star which is called the end of the tail of Leo (35); and Mars, 6½° before the heart of Leo (36).

The radius of the great circle was divided above into 25 units. Measured by these units, the sizes of the epicycles are as follows. In Saturn the radius of the first epicycle consists of 19 units, 41 minutes; the radius of the second epicycle, 6 units, 34 minutes. In Jupiter the first epicycle has a radius of 10 units, 6 minutes; the second, 3 units, 22 minutes. In Mars the first epicycle, 5 units, 34 minutes; the second, 1 unit, 51 minutes (30). Thus the radius of the first epicycle in each case is three times as great as that of the second.

The inequality which the motion of the epicycles brings to the motion of the deferent is called the first inequality; it follows, as I have said, unchanging paths in the firmament. There is a second inequality, on account of which the planet seems from time to time to retrograde and often to become stationary. This happens by reason of the motion not of the planet but of the earth changing its position in the great circle. As the line of sight is directed toward the firmament, the motion of the earth more than neutralizes the motion of the planet, since the former is more rapid than the latter. This regression occurs notably when the earth is nearest to the planet, that is, when it comes between the sun and the planet at the evening rising of the planet.

(33) In *De Rev.* COPERNICUS abandons this doctrine of the fixity of the apsides: *interim quoque et summae absidis locus eccentrici promotus est XIII gradibus et LVIII scrupulis sub non errantium stellarum sphaera, quem credebatur Ptolemaeus eodem modo fixum, at nunc apparet ipsum moveri in centum annis per gradum unum fere* (*Th* 339.7-11); cf. *Th* 351.2-5, 360.2-7.

(34) η Sagittarii (*Th* 139.14).

(35) β Leonis (*Th* 135.12).

(36) Regulus (*Th* 134.23-24).

Near the time of the evening setting or morning rising, on the other hand, by its forward motion the earth makes the observed motion greater than the actual. But when the line of sight is directed toward the planets moving in the opposite direction with equal velocity, they seem to be stationary, since the motions, being in opposite directions, neutralize each other; this regularly occurs when the angle at the earth between the sun and the planet is 120° . In all these cases the inequality is greater to the same degree that the deferent on which the planet moves is smaller. Hence the inequality is smaller in Saturn than in Jupiter, and again greatest in Mars, in accordance with the ratio of the radius of the great circle to the radii of the deferents. The inequality attains its maximum for each planet when the line of sight to the planet is tangent to the circumference of the great circle. In this manner do these three planets move.

In latitude they have a twofold deviation. While the circumferences of the epicycles remain in a single plane with their deferent, they are inclined to the ecliptic at an angle equal to the inclination of their axes, which do not, as in the case of the moon, revolve, but are directed always toward the same points of the heavens. Therefore the intersections of the deferent and ecliptic (these points of intersection are called the nodes) occupy eternal places in the firmament. The node where the planet begins its ascent toward the north is, for Saturn, $8\frac{1}{2}^\circ$ behind the star which is called the eastern in the head of Gemini (37); for Jupiter 4° before the same star; and for Mars $6\frac{1}{2}^\circ$ before Vergiliae (38). When the planet is at this point and its diametric opposite, it has no latitude. The greatest latitude, which occurs in these planets at the quadratures, varies considerably. For the inclination of the axes and circles, changing at the nodes, seems to increase, and becomes greatest when the earth is nearest to the planet, that is, at the evening rising of the planet. At that time the inclination of the axis is, for Saturn $2\frac{2}{3}^\circ$, Jupiter $1\frac{2}{3}^\circ$, and Mars $1\frac{5}{6}^\circ$ (39). On the other hand, near the time

(37) Castor (*Th* 132.28-29).

(38) *Th* 132.5-6.

(39) S: *dextante*; V: *sextante*. PROWE adopted the reading of V; but *sextante* is clearly impossible, for the following sentence of the text states that the inclination diminishes in the case of Mars by $1\frac{2}{3}^\circ$.

of the evening setting and morning rising, when the earth is at its greatest distance from these planets, the inclination is smaller, for Saturn and Jupiter by $1/5^{\circ}$ (40), and for Mars by $1\ 2/3^{\circ}$ (39). Thus this inequality is most notable in the greatest latitudes, and becomes smaller as the planet approaches the node, increasing and decreasing equally with the latitude. The motion of the earth in the great circle also causes the observed latitudes to change, its nearness or distance increasing or diminishing the angle of the observed latitude, as mathematical analysis demands. This motion in libration occurs along a straight line, but can be derived from two circles. These are concentric, and the one carries the revolving poles of the other. The lower circle, in the direction opposite to that of the upper, and with twice the velocity, carries the poles of the circle which serves as deferent to the epicycles. The distance from these last poles to the poles of the circle halfway (41) above is equal to the distance from the poles just mentioned to the poles of the highest circle. So much for Saturn, Jupiter, and Mars and the orbits which enclose the earth.

VENUS

There remain for consideration the motions which are included within the great circle, that is, the motions of Venus and Mercury. Venus has a system of circles like the system of the superior planets (42), but the arrangement of the motions is different. The deferent revolves in nine months, as was said above; the greater epicycle also revolves in nine months; by their composite motion the smaller epicycle is everywhere brought back to the same path in the firmament; and the higher apse is at the point(33)

(40) These angles of inclination are altered slightly in *De Rev.*

		<i>Commentariolus</i>	<i>De Revolutionibus</i>
Saturn	greatest	$2^{\circ}\ 40'$	$2^{\circ}\ 44'$ (<i>Th</i> 422.10-11)
	least	$2^{\circ}\ 28'$	$2^{\circ}\ 16'$
Jupiter	greatest	$1^{\circ}\ 40'$	$1^{\circ}\ 42'$ (<i>Th</i> 422.7-8)
	least	$1^{\circ}\ 28'$	$1^{\circ}\ 18'$
Mars	greatest	$1^{\circ}\ 50'$	$1^{\circ}\ 51'$ (<i>Th</i> 421.22-25)
	least	$0^{\circ}\ 10'$	$0^{\circ}\ 9'$ (<i>Th</i> 421.31-422.1)

(41) V : *mediale*; S : *mediate*. Before S was known, CURTZE emended V to *immediate*, which PROWE prints. But S is undoubtedly correct.

(42) In *De Rev.* COPERNICUS employs for Venus two eccentrics; the larger, outer eccentric which carries the planet has for its center a point which revolves on the smaller eccentric (*Th* 368.23-29).

where I said the sun reverses its course. The period of the smaller epicycle is not equal to that of the deferent and greater epicycle, but has a constant relation to the motion of the great circle. For one revolution of the latter the smaller epicycle completes two, so that whenever the earth is in the diameter drawn through the apse, the planet is nearest to the center of the greater epicycle; and it is most remote, when the earth is at a quadrant's distance from the points just mentioned. The smaller epicycle of the moon moves in very much the same way with relation to the sun. The ratio of the radius of the great circle to the radius of the deferent of Venus is 25 : 18; the greater epicycle has a value of $\frac{3}{4}$ of a unit, and the smaller $\frac{1}{4}$ (30).

Venus seems at times to retrograde, particularly when it is nearest to the earth, like the superior planets, but for the opposite reason. For this regression of the superior planets happens because the motion of the earth is more rapid than theirs, but with Venus, because it is slower; and because the superior planets enclose the great circle, whereas Venus is enclosed within it. Hence Venus is never in opposition to the sun, since the earth cannot come between them, but it moves within fixed distances on either side of the sun. These distances are determined by tangents to the circumference drawn from the center of the earth, and never exceed 48° in our observations. This is the furthest point of Venus' motion in longitude.

Its latitude changes for a twofold reason. The axis of the deferent is inclined at an angle of $2\frac{1}{2}^\circ$, and the node whence the planet turns north is in the apse. However, the inequality which arises from this inclination, although in itself it is one and the same, appears twofold. For when the earth is on the line drawn through the nodes of Venus, the motions in latitude are seen as up-and-down movements along perpendiculars, and are called reflexions. The real motions in latitude appear as inclinations of the deferent, and are called declinations (43); these are seen when the earth is at a quadrant's distance from the nodes. But in all other positions of the earth both latitudes mingle and are combined; one exceeds the other, and by their likeness and

(43) The position of *naturales* in this sentence is somewhat uncertain.

difference they are mutually increased and eliminated. The inclination of the axis is as follows : it has a motion in libration related, not as in the superior planets to the nodes, but to certain other movable points which perform annual revolutions of their own about the planet. Hence whenever the earth is opposite the apse of Venus, this motion in libration attains its maximum, wherever the planet may then be on the deferent. Therefore if it is then in the apse or diametrically opposite, it will not completely lack latitude, even though it is then in the nodes. From this point the libration decreases while the earth moves through a quadrant of a circle from the previously mentioned position, and the point of greatest deviation with a like motion moves an equal distance from the planet. Here no trace of the deviation is found. Then the libration produces a resumption of the deviation; and as the initial point descends from north to south and constantly increases its elongation from the planet according to the distance of the earth from the apse, the planet arrives at the region which previously was south. Now, however, by the law of opposition it becomes north and remains so until the circle is completed and the limit of libration is again reached, where the deviation again becomes greatest, and at the same time equal to the initial deviation. The other semicircle is traversed in like manner. For this reason this latitude (which is usually called the deviation) never becomes a south latitude. In the present instance also it seems reasonable that the motions should take place on two concentric circles with oblique axes, as I explained in the case of the superior planets.

MERCURY

Of all the orbits in the heavens the most remarkable is that of Mercury, which traverses almost untraceable paths so that it cannot be easily studied. A further difficulty is the fact that the orbit is generally invisible in the rays of the sun and that the planet can be observed for a very few days only. Yet Mercury too will be understood, if a person of unusual talent attack the problem.

Mercury, like Venus, has two epicycles which revolve on the

deferent (44). The periods of the greater epicycle and deferent are equal, as in the case of Venus. The apse is located $14\frac{1}{2}^{\circ}$ behind Spica Virginis (33). The smaller epicycle revolves with twice the velocity of the earth; by contrast with Venus, whenever the earth is above the apse or diametrically opposite to it, the planet is most remote from the center of the greater epicycle; and it is nearest, whenever the earth is at a quadrant's distance from the points just mentioned. I said above that the deferent of Mercury revolves in three months, that is, 88 days. I have divided the radius of the great circle into 25 units; the radius of the deferent of Mercury contains $9\frac{2}{5}$ of these units. The first epicycle contains 1 unit, 41 minutes; the second epicycle is $\frac{1}{3}$ as great, that is, about 34 minutes (30).

But this combination of circles is not sufficient here as with the other planets. For when the earth passes through the above-mentioned positions with respect to the apse, the planet appears to move in a much smaller path than is required by the system of circles described above; and in a much greater path, when the earth is at a quadrant's distance from the points just mentioned. Since no other inequality in longitude is observed to result from this, it must be explained by a certain approach to and withdrawal from the center of the deferent (45) along a straight line. This motion requires two small intersecting circles with axes parallel to the axis of the deferent. The distance from the center of the greater epicycle or of the whole epicyclic structure to the center of the small circle which passes through the center of the greater epicycle is equal, from axis to axis (46), to the distance from the center of this small circle to the center of the outer one. This distance has been found to be $14\frac{1}{2}$ minutes (47) of one unit of the 25 by which I have measured the relations of all the distances. The motion of the outer small circle completes two revolutions

(44) In *De Rev.* COPERNICUS employs for Mercury, as for Venus, two eccentrics; and by adding an epicycle (*Th* 376.31-377.8) he retains the motion in libration described later in this section.

(45) V: *centri orbis*; S: *a centro orbis*. PROWE accepts V, although S is to be preferred.

(46) Both MSS read *asse*, which CURTZE changed to *axe*. PROWE prints *axe*, which has been incorporated with some hesitation in the translation.

(47) V: *minutibus 24 et medio*; S: *minut. 14 et medio*. PROWE accepts V, although S is certainly correct.

in a tropical year, while the inner one completes four in the same time with twice the velocity in the opposite direction. By this composite motion the centers of the greater epicycle are carried along a straight line, just as I explained with regard to the librations in latitude. In the aforementioned positions, therefore, of the earth with respect to the apse, the center of the greater epicycle is nearest to the center of the deferent, and it is most remote when the earth is at a quadrant's distance from these positions. When the earth is at the mid-points (48), that is, 45° from the points just mentioned, the center of the greater epicycle approaches the center of the outer small circle, and both coincide. The amount of this recession and approach is 29 minutes (47) of one of the above-mentioned units. This, then, is the motion of Mercury in longitude (49).

Its motion in latitude is exactly like that of Venus, but always in the opposite hemisphere. For where Venus is in north latitude, Mercury is in south. Its deferent is inclined to the ecliptic at an angle of 7° . The deviation, which is always south, never exceeds $\frac{3}{4}^\circ$. For the rest, what was said about the latitudes of Venus may be understood here also, to avoid repetition.

Then Mercury runs on seven circles in all; Venus on five; the earth on three, and round it the moon on four; finally Mars, Jupiter, and Saturn on five each. Altogether thirty-four circles suffice to explain the entire structure of the universe and the entire ballet of the planets.

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(48) The so-called octants.

(49) PROWE's note (vol. II, 201, **) should refer to *De Rev.* V, 28, 32, not VI.