INDETERMINISM VERSUS CAUSALISM

J.R. Croca, R.N. Moreira

Departamento de Física
Faculdade de Ciências
Universidade de Lisboa
Campo Grande, 1700 Lisboa
Portugal
email:croca@fc.ul.pt
rmoreira@fc.ul.pt

Introduction

The development of the quantum paradigm in the first quarter of this century led physics to abandon causality. Until then physicists had believed that their rationality, a heritage of ancient Greek times, enabled them to understand Reality. They had believed that the models they had painfully created were mere approximations of a deeper Reality independent of them. They had believed that, with the progress of science, these models would progressively become better and better. Finally, in a more advanced stage they would be very near to Reality. After the general acceptance of Bohr's paradigm, the way of thinking that in the past had led to so many good results, was completely rejected. With the introduction of the new paradigm, physicists, following Bohr, would have reached the limit of their ability to understand Nature. Therefore causality had to be abandoned because it no longer fitted with the microphysical experimental and theoretical evidence. Heisenberg-Bohr indetermination relations would be the limiting boundaries to our cognition capacity.

The non-causal conception of the world that quantum mechanics proposes is very strange even to quantum physicists. For people not familiar with quantum mechanics it is even harder to accept a world without causality, where the concepts of space and time play a minor role. This fact did not discourage Bohr and his followers. They believed that the development of quantum mechanics would allow mankind to leave behind the archaic concepts of space and time. These concepts were then showing their inadequacy to explain and predict phenomena at the atomic and molecular level. They would be mere crutches we use to translate language the phenomena physicists were dealing with into common. This was unavoidable because the great majority of people were not prepared for the huge jump this paradigm imposes. With the abandonment of causality proposed by quantum physics, mankind would have made an important step ahead. They would be able to overcome the fictitious barriers of space and time. They would achieve omnipresence, the exclusive attribute of divinity.

Notwithstanding the appeal of this proposal, we can not easily understand how it was possible to give up causality. Throughout the history of man the possibility of acquiring the attribute of omnipresence, and its by-products like telekinesis, teleportation and so on, have been seductive promises. What we have to know, right now, is if

Preprint: The final version appeared in Grazer Philosophische Studien, 56(1999)151

mankind in the 20th century, with the appearance of quantum mechanics, has really acquired this capacity, as Bohr and his followers seemed to believe.

This non-causal paradigm was formally introduced into quantum mechanics through non-local Fourier analysis. It was not therefore surprising that the non-local character of quantum mechanics was subsequently derived when its very foundations lie precisely in an intrinsically non-local formalism.

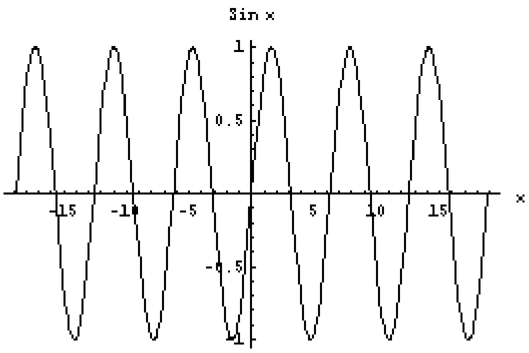
In the present work we set out to accomplish three main objectives. The first is to present the fundamental role played by non-local Fourier analysis played, at both a formal and a conceptual level. The second is to present wavelet local analysis, a new formalism recently developed that restores causality in physics. The third is to present experimental evidence of actual violations of Heisenberg-Bohr uncertainty relations that are mathematically a direct consequence of Fourier non-local analysis. This new experimental evidence can be inserted in a causal framework, allowing us to achieve a more general mathematical formulation for uncertainty relations.

Fourier non-local analysis

When Fourier, during the period between 1807 and 1822, was studying heat conduction, he could not have suspected the fundamental role that the mathematical techniques he was developing would play in the ontological and epistemological discussions on the foundations of 20th century physics.

Fourier thought that it would possible to represent any solution of a differential equation, in particular the heat conduction equation, by a sum (infinite or not) of some particular functions that we call *sine* and *cosine*.

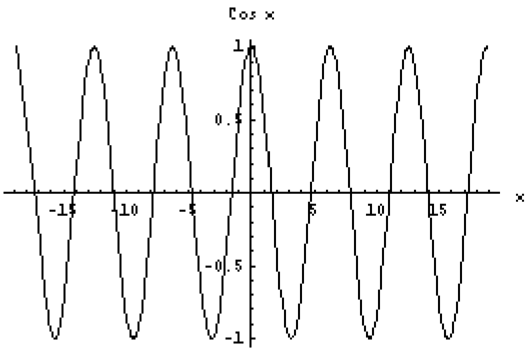
These functions are defined from minus infinity (-∞) to plus infinity (+∞) both in space and in time. This means that if we represent, for example, the function *sin*(*x*) in the below figure, in which argument *x* is either space or time, we obtain:



Graphic of the *sine* function

The figure shows only a very small portion of the function that continues to the left until $x=-\infty$ and to the right until $x=+\infty$.

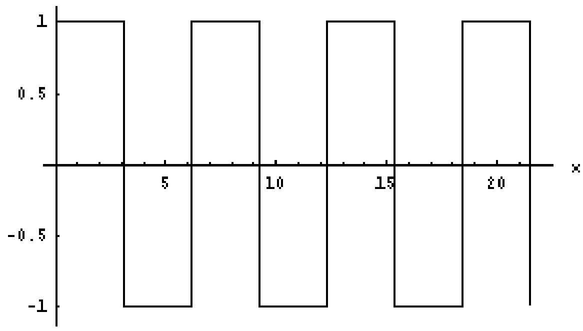
The function *cosine* is a similar function and its graphic representation is given in the figure below.



Graphic of the *cosine* function

These two functions are equal except for a slight change of phase of *π/2*, meaning that the functions are separated by a quarter of a wavelength. One wavelength is the distance between two consecutive peaks.

Let us consider the "squared wave" represented in the following figure:

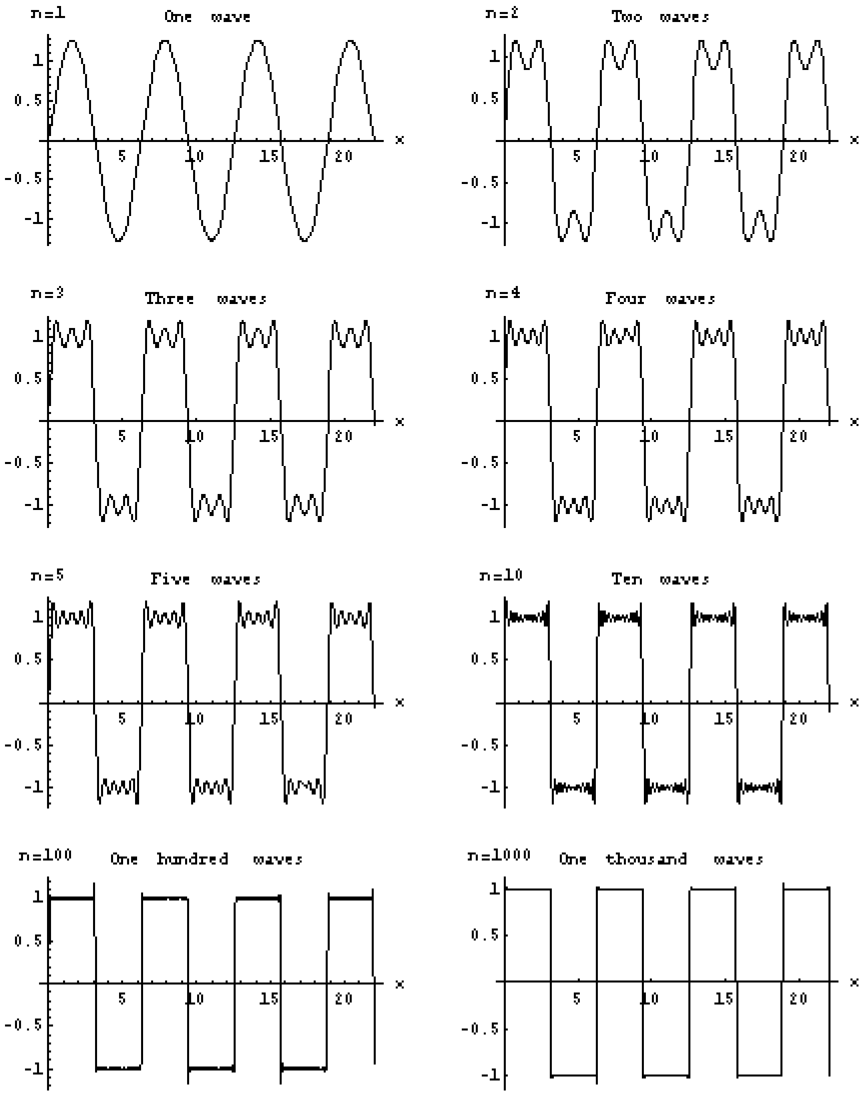


"Squared wave"

The Fourier development of this function is given by:

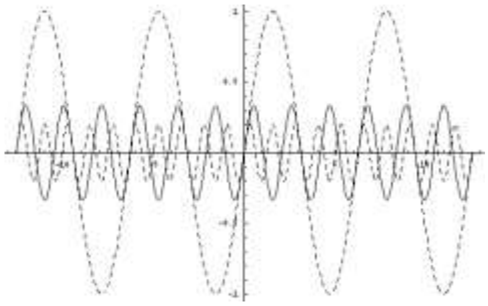
$$f(x) = \frac{4}{\pi}[\sin(x) + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + ...]$$

This expression means that to reconstruct the function *f(x)* (the "squared wave") it is necessary to sum *sin* functions infinitely. The more terms used the better the reconstruction of *f(x)* will be. We will exemplify in the following figure by successive graphics the reconstruction of the function *f(x)* when the number of terms in the Fourier series increases. The cases where the number (n) of elementary waves (*sin/(x)*) used is $n = 1$, $n = 2$, $n = 3$, $n = 4$, $n = 5$, $n = 10$, $n = 100$, $n = 1000$, will be represented.



It can be observed from the figure that when the number of summed waves is one hundred (*n=100*), a fair representation of a “squared wave” is achieved, and when the number of summed waves is one thousand (*n=1000*) we obtain an almost perfect “squared wave”.

It is possible to conclude the following: Looking at the expression of Fourier non-local synthesis it can be observed that there is a basic sinusoidal wave giving a first approximation of the function to reconstruct. It has a wavelength (defined, for example, as the length between two successive peaks) identical to that of the function we want to represent. The amplitude of this first wave is $4/\pi$. The amplitude is the number that multiplies the function *sin(x)*. The second wave summed has a wavelength of a third of the first, and amplitude of a third of it as well. The synthesis continues, summing other waves where the wavelength and the amplitude decrease proportionally to successive odd numbers. To make easier for a reader unfamiliar with mathematical language to understand the above, the next figure represents the first three waves used in non-local Fourier synthesis:



The three first waves used in non-local Fourier synthesis

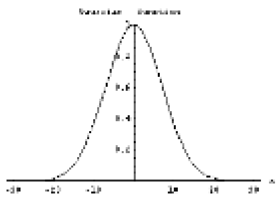
The “squared wave” used is just an example. Obviously, such a wave cannot represent the information that we possess of a microphysical entity. However, this process of non-local Fourier synthesis can be extended to a broader set of functions, giving rise to what is called Fourier’s integral. In this case we replace the infinite sum of discrete waves by an infinite sum of infinitesimal waves.

It is possible to use this new type of synthesis when dealing with non-periodic functions, that is, when the function is only defined in a finite region of the domain (for example, in physical space or in time). This generalization of the Fourier’s non-local synthesis is very useful in physics because the quantum mechanical wave function ψ, which represents all the information we may have about a microphysical entity, may have a limited domain of definition. This is the case when a microphysical particle cannot be detected except in a small region of space and time. A gaussian function, that is, a bell-shaped function, is one example of this case. Purely for interest, we show its mathematical expression:

$$\Psi = A\, e^{-\frac{x^2}{\sigma^2}}$$

In the previous expression A is the amplitude, x represents the argument, for example, space and time, and σ^2 is its variance related with its width at middle height. This function can be used to describe the distribution of the marks of students, the height of the members of a population or the information we have about the possibility of detection of a microphysical particle in a certain region of space and time.

The graphical representation of a gaussian function or gaussian distribution is:



A gaussian function

To reconstruct this function using non-local Fourier analysis it is necessary to integrate, that is, to sum an infinite number of sinusoidal functions (*sines* and *cosines*) the wavelengths and consequently frequencies of which must have a continuous rate of change. It is precisely here where Bohr gets the formal support to introduce the complementarity principle that stands at the very foundations of quantum mechanics.

The principle of complementarity and its ontological and epistemological consequences

At the Conference at lake Como in Italy in September 1927, and at the famous 5th Conference of Solvay during the fall of the same year, Bohr showed that he was deeply acquainted with the epistemological problems posed by quantum formalism. It can be said without fear of inaccuracy that he was the only physicist able to foresee the need to “reconcile” the classical corpuscular and wave conceptions. However, the way he chose to “reconcile” these two classical conceptions was and still is a strange resolution of the contradiction. The answer given by Bohr was to consider the plane harmonic waves, that is, *sines* and *cosines*, as, so to speak, the “arche” of nowadays physics. However it is an “arche” of the phenomena not of the substance, that is, an *“arche”* of the information we can obtain about a microphysical entity which is a result of a complex interaction between the subject and the object. Furthermore, this “arche” exists only potentially. Indeed, it is absolutely necessary to emphasize that, in the spirit of Bohr’s interpretation, these waves, before the interaction with the observer, exist only potentially. Before the interaction with the observer there are no real entities, only a set of possibilities that the act of measurement may make real. It is exactly the non-local Fourier method that allows this interpretation. Indeed, as seen above, when we synthesize a function Ψ representing the information possessed about a microphysical entity, we sum an infinite or finite number of sinusoidal functions. Every one of these functions has a particular wavelength and, therefore, a particular frequency. There is a simple relation between the wavelength (λ) and frequency (υ). Their product is equal to the velocity of propagation (v) of the waves, that is:

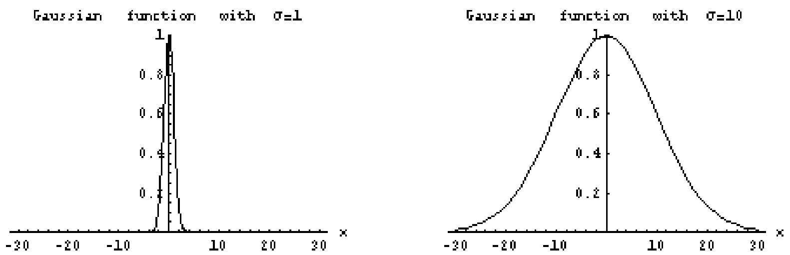
$$\lambda\upsilon = v$$

When the velocity of propagation of the waves is constant, there exists a one to one correspondence between the wavelength and the frequency. This is why *sines* and *cosines* are called in physics monochromatic plane waves or plane harmonic waves. They have a single wavelength and a single frequency. Thus, when we sum a certain number of these waves to synthesize a Ψ wave a problem arises. It is not possible to say that this quantum entity has several frequencies, because this would imply that this quantum entity would simultaneously have several energies. In fact, as it is well known, there is a fundamental relation between frequence and energy in quantum physics expressed in the famous and fundamental relation of Planck-Einstein:

$$E = h\upsilon$$

In this expression E represents energy, h is Planck's constant and υ is the frequency. Thus, different frequencies mean different energies, and this would imply that the quantum entity, being represented by several plane harmonic waves, would possess several energies simultaneously. This would be unacceptable because when a quantum particle is detected it evidences a well-defined energy. In order to solve this problem Bohr introduced a postulate in quantum theory called the *instantaneous reduction of the wave packet*. This means that when we make a measurement only one frequency (energy) will show up. Every other potential plane harmonic waves that build the wave packet turn into nothing. Only one of the energies that the particle potentially possessed will become actual. Among the packet of plane harmonic waves necessary to synthesize the wave function Ψ when we use Fourier's non-local theory, only one of them has the possibility of being actually measured. Every one of the others is reduced to zero instantaneously.

The *principle of complementarity* is, as we said, the touchstone of the interpretation of Bohr of non-local Fourier formalism when applied to quantum phenomena. Indeed, if we consider the gaussian distribution mentioned above, we can consider different distributions with different widths at half height. The width at half height is its main characteristic.



Two gaussian functions with different widths at half height

In the previous figure two gaussian distributions were represented for two different widths (σ=1 and σ=10). The width at half height is smaller in the first than in the second. If the variable x of the function represents the position in space, the first case would represent a particle that can be detected in smaller region of the space then the particle represented by the gaussian at the right. To emphasize this point we can consider two limiting cases. The first would be the case where the width at half height is zero, that is, a function equal to zero everywhere except in the center. This case can be represented by a special function called the *Dirac delta (δ) function*. The second would be the case where the width is infinite, that is, a constant function infinitesimally small everywhere. The first situation would correspond to the case where the position of the quantum entity would be unequivocally known before detection. Conversely, the second situation would correspond to the case where the position of the quantum entity would be completely unknown before detection. In the latter case we could detect the quantum entity with equal probability in any position in space.

In order to synthesize the first function in an acceptable way, non-local Fourier theory tells us that we need to integrate (to sum an infinite number of infinitesimal terms) over all frequencies or, which comes to the same thing, all energies. Thus, in this case we would know the position of the quantum particle with absolute accuracy, but we would know absolutely nothing about its energy. In the case of a free particle (not exposed to any kind of field) the energy of the particle is its kinetic energy:

$$E = \frac{1}{2} m v^2 .$$

In this expression, E represents the energy of the particle, m its mass and v its velocity. Evidently, for particles with the same mass, correspond different momenta (mv) and, consequently, a different velocity (v) to different energies. This implies that we know the position of the particle with absolute accuracy but we know nothing about its momentum.

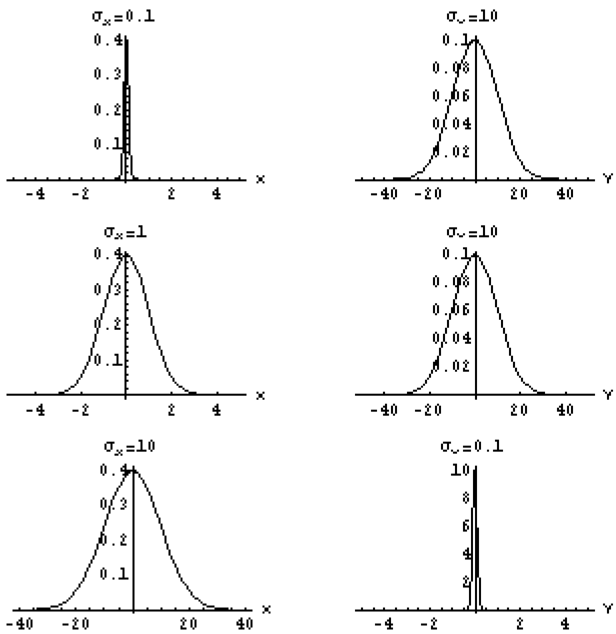
The second situation discussed, where the wave function Ψ has a very small value constant everywhere, represents the quantum entity by a single monochromatic plane wave or plane harmonic wave. This wave is mathematically represented by a *sine* or a *cosine*. A *sine* and a *cosine* have a well-defined wavelength and frequency. This is the reason that they are called in physics monochromatic plane waves or plane harmonic waves. Therefore, the particle would have a well-defined energy ($E = h\upsilon$) and consequently nothing is known about its position.

In contemporary physics, it is customary to say that physicists are squeezed between the wave concept and the corpuscle concept. This is what is meant by the *wave-particle dualism*. Indeed, from a formal standpoint, we use the wave concept as the basic concept. It should be remembered that, as said above, the *"arche"* of contemporary physics are the plane harmonic waves. In fact, these have been the formal "bricks" with which quantum theory is built.

The plane harmonic wave Ψ is the simplest mathematical form for the wave function, with a single frequency, a single wavelength and a single period. Because plane harmonic waves exist from minus infinity to plus infinity both in space and in time, we are induced to apart ourselves from the classical concept of a particle or material point with a well-defined position in space and in time. On the contrary, when we deal with a special function that is null everywhere except at a single point, we are dealing with something that we can easily associate with the classical concept of a material point, with a well-defined position both in space and in time. However, this is something completely different from an ordinary wave, and we know from non-local Fourier theory that we need to integrate over all the frequencies, that is, we must use in this infinite sum all plane harmonic waves to reconstruct this function Ψ. Therefore, it is hard to associate the information we possess on the quantum entity with the classical concept of wave. But, even for this case, for which we can predict, before the measurement, with the greatest possible accuracy, the position of the particle, we know absolutely nothing about its energy and, consequently, its velocity. This fact implies that, even in this case, a quantum entity can not possess a real position and a real velocity before the measurement. It is the measurement that makes actual one of these potentialities. This show how distant we are from the classical concept of particle for, which there is simultaneously well defined both position and velocity. This is due to the fact that, in quantum physics, the basic idea is the concept of wave and the notion of particle is a derived one.

Within this formalism we must conclude that if we want to emphasize the corpuscular character of a quantum entity, by reducing the region of space where it can be detected, we distance ourselves from an wave conception of this quantum entity and vice-versa.

This is the very puzzling essence of the *wave-particle dualism* when we want to translate it in classical terms. It is a mathematically direct consequence of non-local Fourier theory where there is nothing except waves.



Three gaussian functions and its Fourier transforms

In order to understand what happens between these two limiting cases, we represent in the figure several gaussian functions Ψ with different widths. In the left column three gaussian distributions with widths equal to 0.1, 1 and 10 respectively are represented. For each case it is shown in the same row the non-local Fourier analysis of these functions, that is, the value and the weight of the frequencies we need to sum in order to reconstruct the initial function at the left in the same row. When we say that we sum frequencies, it should be understand it as a sum of plane harmonic waves. This is so because to each plane harmonic wave corresponds a different frequency. The non-local Fourier analysis of the previous Ψ functions gives the value and the weight of the different frequencies necessary to synthesize the initial function Ψ, shown at the left in the same row. We call these second functions the Fourier transforms of the first functions. The transform of a gaussian function in ordinary space is another gaussian function in the so called, space of frequencies.

We can conclude that the greater the width of the gaussian function in ordinary space, the smaller the width of the gaussian function in the space of the frequencies, and vice-versa. The Heisenberg-Bohr uncertainty principle is a direct consequence of this fact.

Soon after Bohr became acquainted with Schrödinger's work he clearly caught a glimpse of the way to solve the problems posed by atomic phenomena and theories. Schrödinger had previously shown that non-local Fourier formalism could be applied to solve the problems raised by atomic phenomena. Bohr was, from the very beginning, well aware of these fundamental problems. In 1927 he began to write a paper that was published in *Nature* the next year [1]. In that article we can find the following statement: (p. 581)

> *"While energy and momentum are associated with the concept of particles, and hence can be characterized according to the classical point of view by definite space-time coordinates, the period of vibration and wavelength refer to a plane harmonic wave train of unlimited extent in space and time."*

Bohr shows in this statement that he knew from the outset the intricate relation that non-local Fourier formalism imposed between the concepts of wave and particle. The Heisenberg-Bohr *uncertainty principle* is, as already mentioned, a consequence of non-local Fourier formalism. We use this expression rather than Heisenberg uncertainty principle because, although it was Heisenberg who first discovered it, it was Bohr who turned it into a meaningful statement linking it to its *complementarity principle*, the touchstone of the interpretation of quantum formalism. In the case we have been discussing, the *principle of complementarity* means the existence of a complementarity relation between the position (x) and the momentum (mv). That is, the more precisely we want to know before a measurement the position of a quantum entity, the less we can predict its momentum. Moreover, and Bohr is very clear at this point, this complementarity relation is part of a more general and fundamental definition of the *principle of complementarity*. This new definition implies the existence of a complementarity relation between a *space-time description* and a *causal description*. Using kantian terminology we would specify that there exists a complementarity relation between the *a priori forms of sensitivity* and the *a priori forms of understanding*. The *a priori forms of sensitivity* are the concepts of space and time, and the category of causality constitutes, in the domain of science, the main *a priori form of understanding*. We may quote, among many others, another part of Bohr's paper mentioned above, where he is transparent in the defense of this position: (pp. 580-581)

> *"On one hand, in attempting to trace the laws of time-spatial propagation of light according to the quantum postulate, we are confined to statistical considerations. On the other hand, the fulfillment of the claim of causality for the individual light processes, characterized by the quantum of action [Planck's constant h – our comment], entails a renuncia-*

tion as regards the space-time description. Of course, there can be no question, of a quite independent application of the ideas of space and time and of causality. The two views of the nature of light are rather to be considered as different attempts at an interpretation of experimental evidence in which the limitation of the classical concepts is expressed in complementary ways."

The way Bohr reached the *principle of complementarity* has been already studied by one of us in previous works [2][3]. There, the profound influence Høffding had over Bohr is defended. A similar position has been put forward by Jan Faye in several works [4][5][6]. Other authors, like Favrholdt [7][8][9][10][11], hold the opposite view point. For them, the genesis of the *principle of complementarity* would be an exclusive work of Bohr, without any kind of influence external to the own quantum formalism. We cannot accept this position. The resemblance between the thoughts of both is too great to be a simple coincidence. An important question is not answered if we accept this position. Why did Bohr accept and support the idea that Quantum Mechanics was a complete theory? Why did he not accept that this formalism and the theory he built upon it would be only a good approximate description of real phenomena allowing the possibility to go beyond in future? We think that this only becomes comprehensible if it is accepted that the interpretation of quantum formalism achieved by Bohr was in strong consonance with his long-standing and deeper convictions. Bohr recognized the compatibility between quantum formalism and his own former beliefs, which were linked to the main Danish philosophical (anti-hegelian) stream of thought: Poul Martin Møller, Søren Kierkegaard and Harald Høffding. The latter had been Bohr's professor of philosophy, a friend of Bohr's father and finally a friend of Bohr himself. We shall not go deeper into this point, which has been studied in other works, but we cannot resist quoting a fragment of Høffding's book [12] *La relativité philosophique* that shows a close agreement between the two. Bohr read this book and made an approving comment on the Danish edition's title. In this book Høffding speaks copiously about physics, one possible reason to attract Bohr's attention. Here is the fragment: (pp. 197-199)

> "Continuity and discontinuity are correlatives that supplies each other. They designate different points of view and different operations; the history of sciences shows how both one and the other take the lead, but in such a way that the struggle between them starts afresh always. No one shed a more clarifying light upon them than Henri Poincaré when he says: This struggle will go on as long as there is science, as long as mankind thinks, because it is due to the result of the two irreconcilable needs of the human spirit from which that same spirit cannot strip itself unless it ceases to exist, the need to understand and we are not able to understand but that which is finite, as well as the need to see and we are not able to see except an extension of that which is infinite."

The impossibility of simultaneously *seeing* and *understanding*, can be identified with the impossibility of obtaining simultaneously a *space-time description* and a *causal description*. Thus, the concept of complementarity as Bohr conceived it can be easily found in Høffding. This one thought he had found an irrational residue that human thought would never be able to transcend and Bohr, following him, believed that in quantum physics he had also discovered an insurmountable irrational residue.

Those authors who claim that Bohr was led to his *principle of complementarity* only by physical reasons alone, must explain also why he thought in extending it beyond physics. An article by L. Kay [13] is very revealing because she shows Bohr trying to extend his *principle of complementarity* to biology. A young physicist at the time, Max Delbrück left physics and embraced biology under the influence of Bohr. He wanted to find a *principle of complementarity* in biology. However, in this case there was no mathematical formalism. Consequently, Bohr never achieved a precise expression for this hypothetical principle. The best he could do was to enunciate it in a very broad and general way. It would be something like this: beyond a certain limit, if we want to know more of the secrets of life, then we need to destroy life itself. Kay emphasizes that Max Delbrück's epistemological research program in biology failed disastrously when in 1953 two researchers, Jim Watson and Francis Crick discovered the structure of DNA. This discovery, to a certain extent, reduces the study of life to chemistry and physics. This attempt from Bohr clearly proofs how he profoundly believed that his *principle of complementarity* ought to be applied to all phenomena.

Bohr's interpretation of quantum formalism which is plainly supported by a fundamentalist interpretation of non-local Fourier theory deeply influenced 20th century physics. This fact enable us to say that, in the domain of physics, we can name the 20th century *"Bohr's century"*, in the same way as we call the 17th century *"Newton´s century"*.

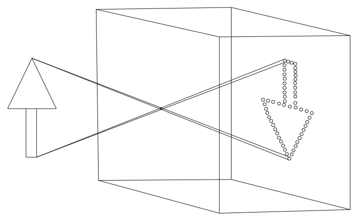
Recently, new advances, both formal and empirical, have shed a new light on the very foundations of physics. We will try to report the way these new data have been progressively emerging at the end of the 20th century. To achieve this we will recall the resolution limit of optical systems. This limit is itself a consequence of non-local Fourier theory.

Theoretical limits for the resolution of optical systems

Optical systems dedicated to amplifying images, like, for instance, the microscopes, have a fundamental theoretical limitation. At first sight this may seem paradoxical. Indeed, people with little knowledge of physics, believe that the capacity of a microscope to amplify an image has no limits. However, essentially the way that leads to this conclusion is very simple: Let us consider an object that we want to amplify. Using a microscope we can enlarge the image to obtain an amplification of, for example, one hundred times. This can be easily achievable with an ordinary microscope. Next we use the amplified image and with an identical microscope have it augmented another one hundred times. This second image corresponds to an overall amplification that is the product of the two amplifications, that is, ten thousand times. Following this procedure we could have any degree of amplification we would like. However, and unfortunately, it is not quite so!

Every optical imaging system possesses two kinds of limit. One, the easiest, is the result of the natural deterioration of the image in successive amplifications. After a certain number of them the image irremediably deteriorates due to the imperfection of the lenses and other natural causes that contribute to a general loss of image quality. The other was discovered by physicists in the last century. This second kind of limitation they called resolution. This is a basic characteristic of any imaging system, setting theoretical limits for its amplification power; it is much more fundamental, and is a consequence of the wave character of light. To understand this important concept let us see the functioning of one of the most common of optical systems: the photographic camera.

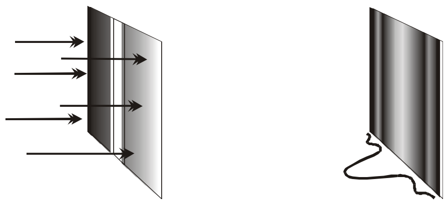
In the following figure we can see a scheme of the pinhole camera that is the essence of the photographic camera.



We can observe that the diameter of the hole defines the resolution of the final image. The smaller the entrance point the higher the resolution. Geometrically, if the pinhole were a mathematical point, every point of the object would produce a single point in the image. In practice, because the physical hole is never a mathematical point, each point of the object produces a circle in the image. This is what we can see in the figure above. But there is a lot more. When the dimensions of the point decrease, we begin to observe the phenomenon of diffraction, and the image points increase instead of diminishing.

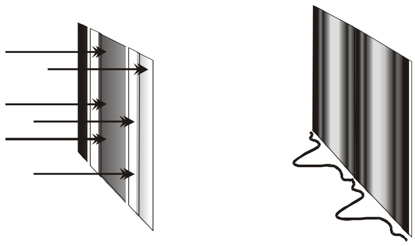
Abbe, a physicist of the last century, showed that the maximum theoretical resolution for these systems is about half the wavelength of the light used in the device. This explains the considerable investment currently being made to produce compact disks systems working with violet light. The wavelength of violet light is about half the wavelength of red light. This implies that the resolution of a reading system using violet light is about double that of a reading system using red light. In practice this means a four-fold increase in capacity. This means that if on a red light CD we can store a two-hour movie, on a violet light CD we can store an eight-hour movie.

We can illustrate this fundamental point with the next figure, where we represent a screen with a vertical slit. Let us consider a parallel beam of light, of a certain wavelength, impinging on the screen with the slit. At the target placed far behind the screen we observe the image of the slit.



Due to the wave character of light the image of the slit, instead of being very sharp, changes into a blurring diffraction pattern with a central primary maximum and a series of secondary maxima to the right and to the left of the main one. In the picture only one secondary maximum was represented for each side. For sake of clarity the intensity distribution observed in the target is represented horizontally at the bottom of it. This diffraction pattern of a rectangular slit corresponds mathematically precisely to the Fourier transform of the rectangular slit.

Let us now consider the case of a screen with two rectangular slits sufficiently far from each other, as shown in the next figure. In this case we observe two different diffraction patterns corresponding to each one of the slits with two primary maxima perfectly separated from each other. Once again it is possible to can calculate the final image as the overlapping of the Fourier transforms of the two slits each considered separately.



We may now ask, what will happen if the distance between the two slits in the screen decreases?

From what we have just said the answer is quite simple. What we will observe on the target is the overlapping of the two different diffraction patterns where the two main maxima merge into a single one. When the distance between the silts is lesser than a certain value the two maxima are no more distinguishable. This represents the resolution limit due to the wave nature of the light. The situation is represented in the following figure.

Abbe was able to show that this limit corresponds to about half a wavelength of the impinging light on to the screen. Thus, below this limit we can no longer distinguish the two slits in the final image. This is, has been said, a consequence of the wave character of light and is mathematically described by non-local Fourier formalism.

Now we can understand why those theoretical results, with profound technological consequences, are a direct result of non-local Fourier formalism, and consequently profoundly connected to Bohr's *complementarity principle*.

Experimental violation of the maximum resolution limit for Fourier systems

We have just concluded that the maximum theoretical resolution limit of half wavelength for an optical system is deeply connected with the very foundations of quantum mechanics. Therefore, we could be led to admit that it is an essential feature of Nature. In this sense we could never go beyond it. However, Nature is much more complex than any human theory no matter however refined and complete it could seem. There are no complete definitive theories, even if some thinkers of the past and even of the present claim the contrary. Scientific theories are man made constructions resulting from social and cultural environment where he is immersed. Theories are always conceived to deal with the experimental evidence we can access with the empirical and theoretical instruments available at the time. In this context theories do not rule natural phenomena. They are always rough approximations that describe with more or less accuracy certain aspects of Nature.

The history of science teaches us that this misunderstanding has been very common among the thinkers of the past and even among some of the present. Dominant theories lead to conclusions that seem completely correct and general. However, some marginal experiments, at first sight not connected to any theoretical structure, lead us to empirical results that not only violate the established theories but also go beyond them, opening new universes of possibilities.

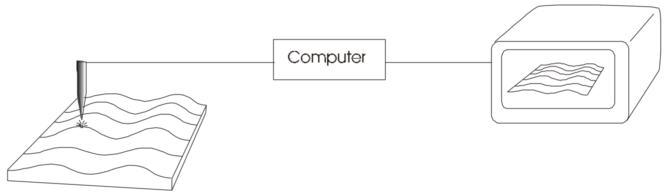
In the history of science in general and the history of physics in particular, there are many examples of practical apparatus used in everyday life which way of working was not clearly explained by any existing theories at the time. This is the case, for instance, of the steam engine. The first worthwhile theoretical formulation only appeared half a century after the invention of these engines. This is also the case of the microscopes not based on the non-local Fourier relations that are nowadays in common use in scientific and technological laboratories. In the following we shall present this new evidence that seems to go against the established paradigm.

A new generation of microscopes

In the middle of the eighties IBM laboratories produced a new type of microscope with an experimental resolution largely better than Abbe's half-wavelength maximum theoretical limit. This was the scanning tunneling electron microscope developed by Binning and Roher [14], two researchers who won the Nobel Prize for the discovery.

Shortly after, the principle of this microscope was extended to the optical domain. At the present moment there are many types of optical microscopes that do not work within the Fourier regime. The practical resolution of some of them is of about 1/1000 of a wavelength, five hundred times better than the limit imposed by Abbe's criterion based upon non-local Fourier formalism!

In the next figure we represent schematically one type of this microscope in order to show how they work. The microscope chosen is the non-Fourier scanning optical microscope developed by Pohl and his group [15].



Basically this microscope has a sensor or light detector, a scanning system that controls the position of the sensor, and a computer with a display device.

The light detector is in general made of a very thin optical fiber with the tip much smaller than the diameter of a human hair. The light collected by the tip of the needle is directed to a large electronic detector that converts the light intensity into an electrical pulse. In some cases, instead of the optical fiber, the sensor extremity can be just a simple very small solid state detector converting directly the light into an electric pulse. In any case the light that strikes the detector is converted into an electric pulse that feeds the computer.

The scanning system, not shown in the figure, is commonly composed of a cantilever which arms are made of piezo-electric quartz crystal. These crystals, very common, for example, in modern cigarette lighters, have a very interesting property. When they are compressed produce an electric current. This is the reason why they are called piezo-electric. It is this current that produces the spark in the lighter. These devices work reversibly, that is, when we apply an electric current to them they shrink or expand according to the field applied. The electrical field applied to the arms of the scanning device controls the position of the tip of the sensor allowing a complete scanning of the whole sample.

The computer receives the electrical pulse from the sensor and, after a suitable processing, a final amplified image of the sample appears on the display. This image is the result of the following process: The sample is illuminated and its points diffuse light in all directions. The tip of the sensor positioned over one point of the sample collects some of the diffused light, and transforms it into an electric pulse proportional to the light intensity. The light intensity captured depends on the distance between the sensor tip and the surface of the sample, and also on the collecting area of the sensor tip. Thus, during the scanning the computer records the variation of light intensity in a scanning line. Scanning successive lines over the whole sample we finally obtain the desired amplified image. For example, if only one point of the sample diffuses light, in the image shown on the display one observes a continuous uniform surface with only one discontinuity. This discontinuity represents the enlarged image of that single point.

The experimental resolution of the apparatus depends on the dimensions of the sensor tip, the accuracy of the scanning device (better for smaller steps), and the minimum distance between the sample and the sensor's extremity (the smaller the better).

We would like to emphasize that in these instruments, known as second-order instruments, the final image is the product of an "intelligent" treatment of the image. The well-known CAT that produces 3D radiological images is an instrument of this type. The sensors collect the intensity of the X radiation diffused by the patient, and convert it into an electrical impulse. These impulses are fed to a computer, and after a relatively complex treatment a 3D image of the patient appears in the display.

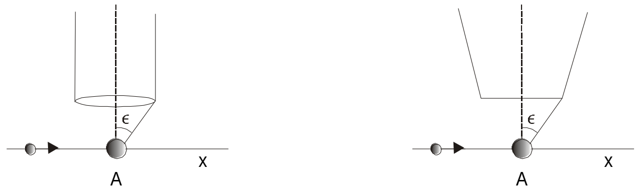
By contrast, in first-order instruments, for example, an ordinary microscope or a magnifying lens, both Fourier systems, the final image results from a simpler and more direct treatment of the information.

Experimental violation of Heisenberg-Bohr inequality

Every measurement is always the result of an interaction between the measuring apparatus and the system we want to analyze. The interaction always produces a perturbation in the system. Let us consider a physical system **A**, a microscopic particle for instance, and suppose that we want to know its position and velocity. To accomplish this goal we need to interact, even minimally, with the system. This interaction modifies the system, so its position and velocity change in an unpredictable way. The uncertainties in the position and velocity of the small particle **A** are such that its product is bigger than or at least equal to a certain value. This minimum value is Planck's constant (h). In other words, after the interaction the particle moves in such a way that its final position cannot be known exactly. We can only say that the particle will be in a certain region of space, the dimension of which corresponds precisely to the uncertainty in its position. The bigger this region, the greater the uncertainty in its position. The same happens with velocity because the interaction also changes it in an unpredictable way.

In order to show that Heisenberg-Bohr uncertainty relations are really violated in this new type of scanning optical microscope we must predict, *before the interaction*, what, *after the interaction*, will be the values of the uncertainties in the position and velocity of a microscopic particle **A,** and show that this product is less than Planck's constant.

In order to facilitate the demonstration let us consider in parallel the cases of the usual Fourier microscope and the new non-Fourier microscope. In the following figure we show detection region of both microscopes.



Common Fourier Microscope New Microscope

As we can see in the figure, the photon moves horizontally and strikes (illuminates) the microscopic particle **A**. We want to predict the uncertainties in the position and in the momentum of that particle. We should recall that the momentum of a particle is the product of its mass and its velocity. When the photon strikes the microscopic particle transfers to it some of its momentum. After being diffused the photon may be caught by the microscope. The calculation for the uncertainty of the momentum of particle **A** after the interaction with the photon can be done in several ways. These can be found in any quantum mechanics textbook. Each author performs the calculation considering more or less factors but all them arrive at the same final expression for the uncertainty of the momentum:

$$\Delta p_x = 2\frac{h}{\lambda}$$

In the last expression, h is once again Planck's constant and λ is the associate wavelength of the microparticle **A**.

The value for this uncertainty is exactly the same for the new non-Fourier microscope. Before being captured by the microscope the photon behaves like a corpuscle in both cases.

The uncertainty in the position for the usual Fourier microscope is calculated using Abbe's maximum resolution criterion which is, as we already know, half a wavelength:

$$\Delta x = \frac{\lambda}{2}.$$

We should recall that this result is due to diffraction and is therefore a consequence of the wave character of light.

This experiment is called in all quantum mechanics textbooks Heisenberg's microscope, and constitutes a paradigmatic example of Bohr's complementarity principle. First, during the photon-microparticle interaction, the photon behaves like a corpuscle. After the interaction, when the photon enters the microscope, it behaves like a wave according to Fourier optics and gives rise to a diffracted spot that represents the image of the particle **A**. Therefore, in this experiment, we have a paradigmatic demonstration of the complementary character of the photon: in some situations it behaves like a particle and in other situations it behaves like a wave.

The product of both uncertainties leads to

$$\Delta x \Delta p_x \geq h ,$$

which is the mathematical expression of the Heisenberg-Bohr uncertainty relations. The equality corresponds to the ideal situation referred above. In real situations the accuracy of measurements will always be worse, and the equality turns into an inequality.

Let us now see what the value of the uncertainty in the position will be for the new non-Fourier microscope. In this case, for the optical scanning microscope represented in the above figure, we do not have any theoretical prediction for the existence of a limit. However, Pohl and his group reached experimental resolutions of approximately

$$\Delta x = \frac{h}{50} .$$

Therefore, the product of both uncertainties $\Delta x \Delta p_x$ leads, in this case, to

$$\Delta x \Delta p_x = \frac{1}{25} h .$$

This corresponds to a violation of the Heisenberg-Bohr relations by a very significant factor of 1/25.

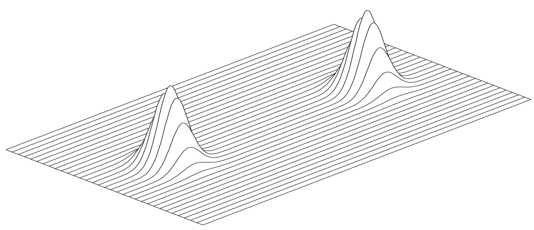
We should mention that in this proof of the experimental violation of the Heisenberg-Bohr relations, we have used a scanning optical microscope with a relatively low experimental resolution. It is possible, at the present moment, to achieve resolutions of about 1/1000. We chose a lower resolution microscope in order to make the demonstration simpler. With this microscope the reasoning that leads to the proof of the violation of the Heisenberg-Bohr relations is, in fact, very similar to the usual one.

The above demonstration, presented to the scientific community before, [17] seems able to resist any argument. However, it must be submitted to other experimental tests. In fact, what we are discussing here is not a minor point, but a fundamental change in an entire vision of the world, a change of paradigm. As we know, it will be not a single experiment that will induce the scientific community to renounce to such a so well-established paradigm. The standard quantum paradigm has been dominant since the twenties. Thus, in order to go beyond it, it is necessary to gather evidence from several experimental and theoretical sources that may lead to the same result.

To pursue this goal, we must call attention to a new mathematical development, which is the wavelet local analysis [18].

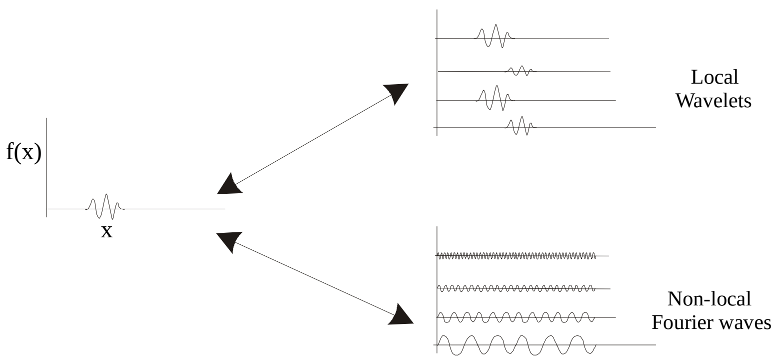
Wavelet local analysis

Using non-local Fourier analysis, as seen before, we were able to reconstruct a large class of functions with unlimited accuracy. However, this analysis is a non-local one, that is, its fundamental “bricks” are unlimited waves, either in space or in time. We called attention to this point at the beginning of this text. When we want to represent the motion of a particle it is necessary to change the phases and amplitudes of the fundamental plane harmonic waves in such a way that they only interfere constructively in the region of space corresponding to the next position of the particle. The non-local interpretation comes from the way we mathematically represent quantum entities. Let us consider two correlated particles, as shown in the figure. When we change the position of one of them the position of the other changes simultaneously, because for correlated particles the infinite waves that potentially describe them are precisely the same. This is the reason why they are sometimes said to be non-separable entities.



If instead Fourier non-local analysis we use wavelet local analysis, we can easily represent a particle mathematically as well. In this case we can reconstruct a function representing a particle without gathering information contained in all space and time. Thus, when a particle moves we only need to change locally the relative position of the wavelets involved.

The following figure represents schematically how the two approaches work.



Both analyses allows us to reconstruct the function *f(x)* representing the particle. However, while non-local Fourier analysis uses mathematical entities existing in whole space and whole time, wavelet analysis uses mathematical entities existing only locally.

Wavelet analysis emerged to solve certain technological problems. This is the case, very common nowadays, of compression of information that we want to save or transmit. For example, if we want to record a movie in digital form it is possible that more than one disk will be required. Using wavelet analysis compression algorithms the same information can be stored on only one disk. This situation has been seen before in the past. Instruments developed to solve particular technological problems have shown themselves very useful in theoretical domains.

Since the establishment of Bohr's paradigm, in 1927, the scientific community tacitly accepted the rejection of the concepts of space and time as fundamental concepts. To tell the truth, we must admit that the majority of physicists use quantum mechanics as a mere tool without being aware of the epistemological implicated problems. Even if in the bohrian quantum mechanics we cannot ascribe any physical reality to the microphysical entities before the measurement, the majority of physicists, even those who use quantum mechanics as a mere working tool, continue to believe in the physical reality of protons, atoms and so. After Bohr, the concepts of space and time, are archaic fictitious concepts. We just use them because they allow us to express ourselves in classical terms but they are not adequate to describe Nature at a more fundamental level. That is, after Bohr, even space and time are no longer real entities. In fact, in some sense, with quantum mechanics man would acquire the divine attribute of omnipresence. Then, he would be able to act here, in space and time, and, simultaneously, everyplace and in all time. Non-local Fourier analysis is the basic formal argument leading them to this belief.

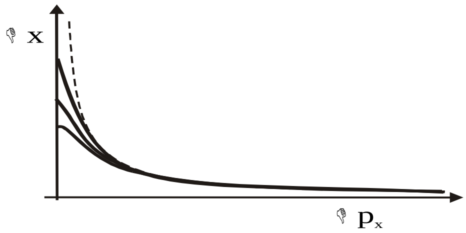
The foundations of Bohr's paradigm are shaking at present as a consequence of two causes: first, the conflict with experimental evidence, and second, the appearance of a local alternative formalism that enables us to describe equally the whole quantum phenomenology in causal terms. Using wavelet local analysis, any physical system can be reconstructed in terms of finite wavelets both in space and in time. We now have the ability to represent a physical system confining ourselves to a limited portion of space and time. The usual description is now a particular consequence corresponding to the case when the width of the basic wavelet is increased as much as we want. The main difference is that now this is not a requirement, but rather something that we can accept or reject according to our needs.

In these terms, local wavelet analysis enables us to recover the lost causality. However, this new causality is much richer than the classical one. It can be extended as much as we like including, as a particular case, the formalism that, until now, has supported the successful non-causal model.

We can mention a particular case where it has been possible to derive a more general set of uncertainty relations than those of Heisenberg-Bohr. Using as a starting point local wavelet analysis instead of non-local Fourier analysis and following step by step Bohr's initial reasoning we are able to derive an expression for the uncertainty relations that contains the usual Heisenberg-Bohr relations as a particular case. These new, more general local relations have the following mathematical expression: [16]

$$\Delta x^2 \geq \frac{h}{\Delta p_x^2 + \gamma^2} ,$$

In this expression the constant γ represents a parameter related to the width of the fundamental wavelets. When the width of the wavelet tends towards infinity γ tends to zero and we achieve the Heisenberg-Bohr relations. In the following figure these new relations are plotted together with the usual Heisenberg-Bohr relations.



In the figure solid lines represent the new relations for three different values of the parameter γ. The dotted line represents the usual Heisenberg-Bohr relations. We can conclude that only in a small region, near the origin, there is a sharp difference between the different lines. This difference enables us to conceive experimental situations where the predictions of both expressions are different. In the remaining domain, the most common case, the different lines coincide, that is, they predict exactly the same experimental results.

Conclusion

It has been shown, even succinctly, that the bohrian non-causal paradigm may be substituted by a different one that restores causality to microphysics. This new paradigm, still under formal development, relies mainly on the local wavelet analysis and in a non-linear approach. In this context, the limits imposed by the new uncertainty relations derived within the new framework are not obstructive barriers for our rational understanding of Reality. They no more represent an irrational residue for our thought. They are mere sporadic empiric obstacles that can, in principle, be overcome by the progress of science. The known experimental evidence points towards a return to a new causality even at the microphysical level. It should be emphasize that it is extremely important to search for other experimental situations showing the limitations and the incapacity of Heisenberg-Bohr uncertainty relations to describe certain aspects of the quantum phenomenology. The restoration of a new causality in microphysics needs a sound experimental basis together with a well-structured mathematical formulation.

Bibliografia

1 – Bohr, N., The Quantum Postulate and the Recent Development of Atomic Theory. ***Nature***, **14**, pp. 580-590, (1928).

2 - Moreira, R., Høffding and Bohr: Waves or Particles, *in Waves and Particles in Light and Matter*, pp. 395-410, ed. Alwyn van der Merwe and Augusto Garuccio, Plenum Press.

3 – Moreira, R., *Contribuição para o estudo da génese do princípio de complementaridade*, PhD dissertation in the University of Lisbon, (1993).

4 – Faye, J., The Influence of Harald Høffding's Philosophy on Niels Bohr's Interpretation of Quantum Mechanics, *Danish Yearbook of Philosophy,* ***16****, pp. 37-72, (1979).*

5 - Faye, J., The Bohr-Høffding Relationship Reconsidered, *Stud. Hist. Phil. Sci.,* ***19****, 3, pp. 321-346, (1988).*

6 - Faye, J., *Niels Bohr: His Heritage and Legacy: An Anti-Realistic View of Quantum Mechanics,* Kluwer Academic Publishers, (1992).

7 – Favrholdt, D., Niels Bohr and Danish Philosophy, *Danish Yearbook of Philosophy*, **13**, Copenhagen, (1976).

8 – Favrholdt, D., On Høffding and Bohr. A Reply to Jan Faye. *Danish Yearbook of Philosophy*, **16**, pp. 73-77, Copenhagen, (1979).

9 – Favrholdt, D., The Cultural Background of the Young Niels Bohr, *Rivista di Storia della Scienza*, **2**, 3, (1985).

10 – Favrholdt, D., Remarks on the Bohr- Høffding Relationship, *Stud. Hist. Phil. Sci.*, **22**, 3, pp. 399-414, (1991).

11 - Favrholdt, D., *Niels Bohr Philosophical Background*, Munksgaard, Copnhagen, (1992).

12 – Høffding, H., *La relativité philosophique*, Alcan, Paris, (1928). This work contains two different books written in Danish by Høffding, namely, *Relation as a Category* and *Totality as a Category*. The work Bohr read was exactly the first one: *Relation as a category*.

13 – Kay, L., The secret of life: Niels Bohr Influence on the Biology Program of Max Delbrück, *Rivista di Storia della Scienza*, **2**, 3, pp. 487-510, (1985).

14 – G. Bennig and H. Rohrer, *Helv. Phys. Acta* 55(1982)726

15 - D.W. Pohl, W. Denk and M. Lanz, *Appl. Phys. Lett*. 44(1984)651

16 – J.R. Croca, *Experimental Violation of Heisenberg Uncertainty Relations*} talk at the International Conference on the Foundations of Quantum Mechanics, The 5th Conference at Oxford. September, 1996; *The limits of Heisenberg's Uncertainty Relations*. talk at the International Symposium, Causality and Locality in Modern Physics and Astronomy - Open Questions and Possible Solutions, Toronto, August 1997; J.R. Croca, A. Rica da Silva and J.S. Ramos *Experimental Violation of Heisenberg's Uncertainty Relations by The Scanning Apertureless Optical Microscope*, to be published.

17 - A. Grossmann and J. Morlet, SIAM *J. Math. Anal.* **3**, 723(1989); *Decomposition of Function into Wavelets of Constant Shape and Related Transforms*, (World Scientific, Singapore, 1980); C.K Chui, *An Introduction to Wavelets*, (Academic Press, N.Y. 1992)

18 - J.R. Croca, in *Fundamental Problems in Quantum Physics*, Eds. M. Ferrero and A. van der Merwe, (Kluwer Academic Publishers, 1995); J.R. CROCA and A. Rica da Silva, *Uncertainty Relations for the Superposition of Monochromatic Finite Pulses*, to be published.